

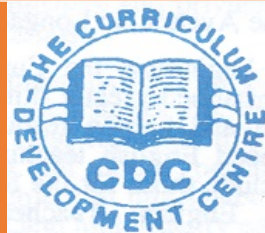


Republic of Zambia  
MINISTRY OF EDUCATION

# **MATHEMATICS II**

## **TEACHING MODULE**

### **FORM 1 TERM 1**



Developed by the Curriculum Development Centre  
Lusaka

2025

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**MATHEMATICS II (STEM)**  
**TEACHING MODULE**  
**FORM 1:TERM 1**

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## **Vision**

Quality, life- long education for all which is accessible, inclusive and relevant to individual, national and global needs.



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## **Preface**

The **Mathematics II Teaching Module** for Form 1 is designed to help teachers prepare and deliver Competence-Based lessons with a comprehensive understanding of Mathematics concepts, fostering a deep appreciation for the role of Mathematics in everyday life and its applications in various fields. This module aims to assist teachers to help learners develop analytical thinking, problem-solving skills, and project based skills through a structured and progressive learning approach. This module prescribes on hands-on activities, inquiry-based learning experience, encouraging learners to explore, experiment, and engage in problem solving reasoning.

This Mathematics Teaching Module for Form 1 aims to help teachers create a stimulating and supportive learning environment where learners can develop a profound understanding of Mathematics. The module help teachers to guide learners who are taking the **Science, Technology, Engineering and Mathematics (STEM) subjects** to grow intellectually and personally by preparing them for professions in **STEM** as well as for higher education by encouraging curiosity, critical thinking, and practical skills.

It is hoped that through this module teachers will inspire learners to explore the fascinating world of Mathematics and appreciate its significance in shaping the future.

Joel Kamoko, (Mr.)  
Permanent Secretary- Educational Services  
**MINISTRY OF EDUCATION**

## Acknowledgements

The development of this Mathematics Teaching Module was a collaborative effort, and we would like to extend our sincere gratitude to the following Directorates, institutions, individuals and subject associations.

Many thanks go to individuals, institutions and organisations that participated in the successful development of this module. These include; the Teachers, Lecturers from Colleges, Public Universities in Zambia. Their valuable insights, expertise, and feedback were instrumental in shaping the content, structure, and overall direction of this module. We appreciate their dedication, time, and effort in helping the Ministry of Education to design and develop a comprehensive and relevant Mathematics II Teaching Module that will help teachers guide learners taking the STEM related subjects in ensuring that they enjoy and appreciate the value of this Mathematics and its application in their everyday life.

We also extend our gratitude to the Zambia Education Enhancement Project (ZEEP) for the financial support and Zambia Educational Publishing House (ZEPH) for the technical support towards the development and finalization of the module.

Last but not the least, I wish to recognize the commitment and hard work of all the staff at the Curriculum Development Centre in ensuring that this module comes to reality is recognised.

Charles Ndakala, (Dr.)  
Director – Curriculum Development  
**MINISTRY OF EDUCATION**

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## HOW TO USE THIS MODULE

To effectively use this Mathematics Teaching Module for Form 1:

- Read and familiarise yourself with the module's content, learning activities, and assessment guides.
- Plan your lessons in advance, using the module's suggested teaching and learning activities.
- Use a variety of teaching methods, including demonstrations, discussions, group work, and hands-on activities.
- Encourage active learning by asking open-ended questions, promoting critical thinking, and fostering problem-solving skills.
- Assess learners learning regularly, using the module's suggested assessment strategies and tools.
- Provide feedback and support to learners, helping them to identify areas for improvement and develop their skills.
- Integrate technology into your teaching, using multimedia resources and interactive simulations to enhance learner engagement and understanding.
- Monitor learners progress and adjust your teaching strategies as needed, to ensure that all learners meet the learning objectives

By following these steps, you can effectively use this Mathematics Teaching Module for Form 1 to support your teaching and promote learning.

## INTRODUCTION

Mathematics, as a fundamental STEM subject, which plays an important role in shaping learners' understanding of the natural world. As learners transition from upper primary to secondary education, it is necessary to provide them with an all-inclusive and engaging introduction to Mathematics. Therefore, the Mathematics Teaching Module for Form 1 is a comprehensive educational resource designed to support the teaching and learning of Mathematics for learners in their first year of secondary education. It covers fundamental concepts and principles of Mathematics, aligning with the 2024 Mathematics syllabus. The module includes topic and subtopic overviews, suggested teaching and learning materials, environment set-up, learning activities, and assessment guides. Additionally, It incorporates teaching methods and strategies like inquiry-based learning, problem-solving activities, group discussions, hands-on activities, multimedia resources, and interactive simulations. The aim is to provide a solid foundation in Mathematics, foster critical thinking and problem-solving skills, and inspire further studies in Mathematics.

## SUGGESTED TEACHING AND LEARNING MATERIALS

The study of Mathematics content requires hands-on experiences, visual aids, and interactive resources to strengthen deep understanding and appreciation. To support the teaching and learning of Mathematics in Form 1, the suggested teaching and learning materials are meant to:

- Enhance learner engagement and motivation in Mathematics
- Develop practical skills and inquiry based thinking
- Promote critical thinking, problem-solving, and analytical skills
- Support differentiated instruction and inclusive learning

The suggested teaching and learning materials are either artificial or natural. By utilising these suggested teaching and learning materials, teachers can create a dynamic and supportive learning environment that promotes academic excellence, creativity, and scientific literacy in Mathematics for Form 1 learners.

## LEARNING ENVIRONMENT SET UP

To create an effective learning environment for teaching and learning Mathematics is important in deepening understanding of the concepts and application in real life context. The learning environment set-up aims to create a safe, inclusive, and engaging space.

- **Natural Environment:** A natural learning environment is a setting where learners explore and learn naturally, often without explicit instruction or formal teaching, such as in school surroundings

- **Man-Made Environment (Artificial):** Man-made learning environments are intentionally designed safe spaces, such as classrooms, laboratories, and libraries, designed for formal instruction, hands-on activities, and games and songs
- **Technological Learning Environment:** Access educational apps, games, and software for learning, including game-based platforms, virtual platforms, and simulations, to engage learners and promote learning

## SAFETY IN THE LEARNING ENVIRONMENT

Safety in the learning environment is a requirement in learning Mathematics, as learners are exposed to potential hazards during hands-on activities. Collaboration between teachers, and learners, is important to create a responsible learning environment that promotes scientific inquiry. Guidelines for maintaining a safe learning environment include, risk identification, personal protective equipment, emergency response plans, storage, disposal, and learner responsibilities. Prioritizing safety minimizes risks, prevents accidents, and ensures a positive learning experience.

## TEACHING METHODOLOGY

The effective teaching methodologies in STEM Mathematics include:

- **Conducting projects:** Demonstrate key principles and encourage curiosity among learners.
- **Collaborative learning:** Pair learners to work together, promoting peer-to-peer teaching, discussion, and problem-solving.
- **Conceptual learning:** Connect mathematical concepts to everyday life, industry, or current events, making learning relevant and meaningful.
- **Differentiated instructions:** Tailor teaching to meet diverse learning styles, abilities, and interests of different learners.
- **Feedback and reflection:** Encourage learners to reflect on their learning, providing constructive feedback to guide improvement.
- **Inquiry-based learning:** Encourage learners to explore, investigate, and discover Mathematical concepts through hands-on activities.
- **Integration of technology:** Use digital tools, simulations, and visualizations to enhance engagement, understanding, and analysis.
- **Problem-based learning:** Present real-world problems or case studies or scenarios, requiring learners to apply Mathematical knowledge to develop solutions.

- **Project -based learning:** Assign open-ended projects, allowing learners to design, conduct, and present research or applications of Mathematical concepts.

By implementing these methodologies, a teacher can create an engaging, inclusive, and effective STEM Mathematics learning environment.

## LEARNING ACTIVITIES

Learning activities are intentional educational experiences aimed at promoting learning, engagement, and achievement among learners. Facilitated by teachers, they help acquire new knowledge, skills, attitudes, and behaviour change. To create an inclusive environment, teachers should use a "hook" or problem posing or scenario or key question or case studies to introduce new learning activities in an interactive and interesting way.

## ICONS USED IN THIS MODULE

This module utilises icons as visual symbols or graphics to represent instructions, enhancing the learning experience for learners. Icons categorize and organise instructions, making navigation easier for teachers.



Key Terms



Assessment



Activity



Tips



Study skills



Summary

## TIME ALLOCATION

The standard minimum learner-teacher contact time for Mathematics at secondary school level is 4 hours per week, translating to Six (6) periods with at least two double periods per week. The duration for a single period is 40 minutes. The contact time at Secondary school level is planned in such a way as to give ample time for practical activities.

## ASSESSMENT

- Formative Assessments: To monitor learner progress, identify areas of improvement, and adjust instruction to meet learner needs.
- Quizzes: Regular quizzes to assess learners' understanding of concepts.
- Tests: Periodic tests to evaluate learners' knowledge and application of Mathematics concepts.
- Class Discussions: Observing learners' participation and engagement in class discussions.
- Group Work: Assessing learners' ability to work collaboratively and contribute to group tasks.
- Projects: Evaluating learners' ability to design, conduct, and present a Mathematics project.
- Presentations: Assessing learners' ability to communicate Mathematics concepts and ideas effectively

### Summative Assessments:

To evaluate learners learning at the end of a lesson, sub-topic, topic or term, and to provide a comprehensive picture of learner achievement.

- **Unit Tests:** Comprehensive tests to evaluate learners' understanding of Mathematics concepts at the end of each unit.
- **Mock Exams:** Comprehensive exams to evaluate learners' overall understanding of Mathematics concepts at the end of the Term.
- **Practical Tasks:** Assessing learners' problem solving skills and techniques through practical exams.
- **Project-Based Assessments:** Evaluating learners' ability to design, conduct, and present a Mathematics project.

## Key Competences

COMPETENCE	DESCRIPTORS
<b>Analytical Thinking</b>	<ul style="list-style-type: none"><li>• Identify patterns</li><li>• Compile data, create mental images and address issues</li><li>• Evaluate solutions</li></ul>
<b>Collaboration</b>	<ul style="list-style-type: none"><li>• Solving puzzle in groups</li><li>• Play with peers to build relationships</li><li>• Participate in and express themselves through play activities</li></ul>
<b>Communication</b>	<ul style="list-style-type: none"><li>• Use mathematical/scientific language in different situations</li></ul>

COMPETENCE	DESCRIPTORS
	<ul style="list-style-type: none"> <li>• Express oneself using different media and symbols</li> <li>• Ask for feedback</li> </ul>
<b>Critical Thinking</b>	<ul style="list-style-type: none"> <li>• Ask and answer simple questions</li> <li>• Classify objects according to their attributes</li> <li>• Manipulate different objects</li> <li>• Solve simple problems in life</li> <li>• Match different things according attributes</li> <li>• Arrange objects according to attributes</li> <li>• Compare similarities or differences between objects</li> <li>• Explore the environment</li> <li>• Differentiate good from bad</li> <li>• Recognize and name items in the environment</li> </ul>
<b>Problem Solving</b>	<ul style="list-style-type: none"> <li>• Make connections/link with the inner world or social environment</li> <li>• Use numeracy patterns and relations to solve problems</li> <li>• Manipulate numbers, shapes and symbols to complete a task</li> </ul>

## TOPIC 1: REAL NUMBERS

### Introduction

**Overview:** Real numbers encompass all the numbers that can be represented on the number line, including rational and irrational numbers. In other words, real numbers can be defined as the union of both rational and irrational numbers. They can be either positive or negative numbers including zero. They are denoted by the symbol “R”. All the natural numbers, decimals and fractions come under this category. These numbers are called real to distinguish them from imaginary or complex numbers. Rational numbers are numbers that can be expressed as a fraction of two integers, whereas irrational numbers cannot be expressed as a simple fraction (e.g.,  $\sqrt{2}$ ,  $\pi$ ). Additionally, the study of real numbers includes exploring their squares, cubes, and patterns. Real numbers play an essential role in everyday life, including in calculations, measurements, scoring and grading, game scores, points, and scientific analysis. Real numbers are also used to represent monetary values, interest rates, and economic indicators in economics and finance. Exploring real numbers is essential for building a strong foundation in arithmetic, algebra, and number theory.

In this topic, the subtopic is types of numbers. Natural numbers, integers, whole numbers, prime numbers, odd numbers, even numbers, composite numbers, squares and square roots, cubes and cube roots; rational and irrational numbers are examples of types of numbers.

### General Competences:

- Analytical Thinking: Evaluate and categorize numbers.
- Creativity and Innovation: Create puzzles and patterns using different types of numbers.
- Critical Thinking: Distinguish between rational and irrational numbers.
- Problem Solving: Solve problems involving square roots and cube roots.



### Key Terms/Vocabulary:

- **Real Numbers:** Numbers on the number line.
- **Rational Numbers:** Numbers expressible as a ratio of two integers (e.g.  $\frac{1}{2}$ , -3, 4, 0.7).
- **Irrational Numbers:** Numbers that cannot be expressed as simple fractions (e.g.  $\sqrt{3}$ ,  $\pi$ ).
- **Square Root:** A number which, when multiplied by itself, gives the original number (e.g.  $\sqrt{16}= 4$ ).
- **Cube Root:** A number which, when multiplied by itself three times, gives the original number (e.g.,  $\sqrt[3]{27} = 3$ ).

## Sub -Topic: Types of Numbers

**Introduction:** Numbers are the building blocks in mathematics and they come in various forms. Understanding the different types of numbers is essential for mathematical operations, problem solving and real world applications.

**Specific Competence:** Use the different types of numbers in real life.



### Learning Activity 1: Exploring Different Types of Numbers

**Introduction:** Real numbers include various types of numbers such as natural numbers, whole numbers, integers, rational numbers, and irrational numbers. They form the basis for arithmetic and algebraic calculations. Real numbers are commonly used in everyday life, from counting objects to physical quantities, such as distance, velocity and acceleration, in science and engineering. Real numbers are also applied in finance and budgeting, inventory management, time keeping and scheduling, scoring and grading, distance and measurement, game scores and points, just to mention but a few. Exploring real numbers is essential for building mathematical foundations, developing problem-solving and critical thinking skills, and fostering curiosity and creativity.

You can introduce real numbers through a mix of visual tools, hands-on activities, and real-life examples. Learners can develop a strong and intuitive grasp of this essential mathematical concept. Here are several engaging activities to help learners explore and understand real numbers. These activities are designed to be hands-on, interactive, and relevant to real-life applications.

#### **Problem Posing**

Find a number that is:

- (a) *A whole number but not a natural number*
- (b) *An integer but not a whole number*
- (c) *A real Number that is irrational*



### Activity 1.1: Research on Numbers

#### Suggested Teaching and Learning Materials to Set up Learning Environment.

- **Artificial Environment:** Books, playing cards, posters
- **Technological Environment:** Computers, tablets, ipads, calculators, phones.

**What to do:** Divide learners into groups. Assign each group to explore on the types of numbers.

**Task:** Ask each group to give examples of the types of numbers explored. Let them present numbers in different ways such as diagrammatic presentation to the class.



**Content Tip:** Ask learners if they found some numbers appearing in more than one category of numbers, let them share their findings about these numbers. Ask them where in real life these numbers are found.



**Skills to be developed:** Research, teamwork and classification skills.

### Activity 1.2: Experiment Using a Die

#### Suggested Teaching and Learning Materials to Set up Learning Environment.

**Artificial Environment:** A Die

**What to do:** Put learners into small manageable groups of mixed abilities.

**Task:** Ask learners in groups to throw a fair six sided die a number of times (say 10 times). Ask them to write down the number of times odd numbers, even numbers, prime numbers, composite numbers appear on top. Let them make presentations to show their findings of the experiment to the class.



**Skills to be developed:** Experimental, classification and collaboration skills.

### Activity 1.3: Magic Square Game

**Suggested Teaching and Learning Materials to Set up Learning Environment.**

**Artificial Environment:** Worksheets

**What to do:** Put learners into small manageable groups of mixed abilities. Come up with worksheets of different magic squares.

**Task:** Ask learners in groups to complete the magic square games, whereby they get the same number when they add across each row, add down each column, and add diagonally. The first group to correctly complete the magic squares wins. Below are the two examples of the magic squares.

7		
	6	4
		5

60	10	150	
	120		50
	70		
	100	40	30

17	24		8	15
		7	14	
	6	13	20	22
10		19	21	3
11	18		2	



**Skills to be developed:** Problem-solving, critical thinking and collaboration skills.

**Activity 1.4: Number Sorting**

**Suggested Teaching and Learning Materials to Set up Learning Environment.**

**Artificial Environment:** Number cards, empty boxes/draw circles on the floor

**What to do:** Provide number cards with different numbers written on them (e.g. 1 – 20, 21 – 40, 41 - 60) to learners in small manageable groups of mixed abilities. Put two empty boxes, one labeled prime numbers and the other composite numbers per group for learners to place their number cards. Alternatively, draw two large circles on the floor for each group labeled as such.

**Task:** Let learners sort the number cards by placing them in their respective boxes or circles in their small groups.



**Content Tip:** Ask them how the two categories of numbers (prime and composite) are related and their use in real life.



**Skills to be developed:** Classification, problem solving and teamwork.

### **Activity 1.5: Number Line Relay Race**

#### **Suggested Teaching and Learning Materials to Set up Learning Environment.**

- **Artificial Environment:** Number cards, flip charts
- **Natural Environment:** Draw number line on the ground using available resources.

**What to do:** Put learners into small manageable groups of mixed abilities. Give each group a large number line drawn on a long sheet of paper or on the ground. Each number line should have a range of rational numbers you have provided. Give each group rational numbers written on number cards. The cards should contain both fractions and decimals.

**Task:** Ask learners to arrange numbers from least to greatest on the number line. The first group to correctly order all their numbers wins.



**Skills to be developed:** Ordering skills, critical thinking and teamwork.

**Conceptualisation:** Make sure the concept of rational and irrational numbers is coming out accordingly. Clearly demonstrate the following numbers and give brief notes on them: Natural number, whole, integers, prime and composite numbers

**Synthesis:** Connect the use of real numbers to real life situations

**Evaluation:** Check if the learner can identify and work with real numbers in many situations. Use a checklist to check the understanding of learners. Ask learners open ended questions to confirm that they can recognize different types of numbers (natural, whole, prime, odd, even, composite, rational, irrational) and apply them in real-life situations.



**Assessment :** Let the learners investigate how different natural numbers and whole numbers are, from integers.



### **Learning Activity 2: Distinguishing Rational and Irrational Numbers**

**Introduction:** Rational numbers can be expressed as fractions, while irrational numbers cannot. Examples include  $\frac{1}{2}$  and  $\sqrt{2}$  respectively. These classifications help in understanding patterns and calculations.

**Problem Posing:** *Can the sum of a rational number and an irrational number ever be a rational number? Justify your answer with an example.*

#### **Activity 2.1: Classifying Numbers as Rational or Irrational**

**Suggested Teaching and Learning Materials to Set up Learning Environment.**

**Technological Environment:** Calculators, phones, tablets, ipads, computers

**What to do:** Put the learners in pairs and provide them with a list of numbers. Ask them to classify the number as rational or irrational.

**Task:** Let learners use calculators, phones, tablets, ipads, computers to confirm decimal expansions (e.g.  $\frac{1}{3} = 0.333\dots$  is rational;  $\pi$  is irrational).



**Content Tips:** Let the learners express  $\frac{3}{8}$  and  $\frac{12}{7}$  as decimals. Decimals such as 1.7142857... that do not terminate and are called infinite decimals on the other hand decimals which terminate such as 0.375 are called finite decimals. Let the learners present recurring decimals correctly.



**Skills to be developed:** Critical thinking classification and ICT skills.

### **Activity 2.2: Fun with Rational and Irrational Numbers**

#### **Suggested Teaching and Learning Materials to Set up a Learning Environment**

**Artificial Environment:** Number cards

**What to do:** Put learners into small manageable groups of mixed abilities. Provide them with number cards with rational and irrational numbers written on them.

**Task:** Have learners sort them in their small groups. The first group to correctly sort the numbers as rational and irrational wins the competition.



**Skills to be developed:** Categorization, critical thinking and teamwork.

**Conceptualisation:** Ensure that the concept of rational and irrational numbers is coming out accordingly. Clearly let learners demonstrate the difference between rational and irrational numbers (e.g. terminating, recurring) and give brief notes on them.

**Synthesis:** Connect the use of rational and irrational numbers to real life situations

**Evaluation:** Check if the learners can distinguish between rational and irrational numbers and apply these numbers in real life situation. Use a checklist to check the understanding of real-life scenarios on rational and irrational numbers.



**Assessment:** Give learners a research project on how rational and irrational numbers can be applied in real life situation. Use the technological environment for this activity.



### **Learning Activity 3: Evaluating Squares and Cubes**

**Introduction:** Squaring a number means multiplying a number by itself or raising a number to the power of two, while cubing means multiplying a number by itself three times or raising a number to the power of three. Understanding square roots and cube roots helps build a strong foundation in Mathematics. They are essential for solving more complex problems involving roots, exponents, and logarithms. Moreover, they enable learners to understand and work with numbers in a more intuitive way, helping them make sense of the world around them. This is essential in geometry, algebra, and practical problem-solving.

**Problem Posing:** *A farmer is planting trees in a square formation, with each tree planted 6 meters apart. If the farmer plants a total of 25 trees, how much area will the trees cover? Use the concept of squares to solve.*

#### **Activity 3.1: Square and Cube Roots (mental math)**

##### **Suggested Teaching and Learning Materials to Set up a Learning Environment**

- **Artificial Environment:** Flip chart, chalkboard/white boards
- **Technological Environments:** Computers, tablets, calculators, phones, ipads

**What to do:** Provide learners with a table of numbers.

**Task:** Ask them to calculate the square roots and cube roots using mental math or any other gadgets (e.g.,  $\sqrt{16} = 4$ ,  $\sqrt[3]{27} = 3$ ).



**Content Tips:** Let learners generate the square numbers (1, 4, 9...) and cube numbers from (1, 8, 27...) so that they understand the square roots and cube roots.



**Skills to be developed:** Analytical thinking computational and ICT skills.

### Activity 3.2: Splitting Whole Numbers into Square Numbers

#### Suggested Teaching and Learning Materials to Set up a Learning Environment

- **Artificial environment:** Number cards

**What to do:** Put learners into small groups of mixed abilities. Give each group, number cards with whole numbers written on them.

**Task:** Ask learners to split those whole numbers into square numbers. For example

$$13 = 9 + 4$$

$$24 = 16 + 4 + 4$$

$$35 = 25 + 9 + 1$$

‘Some whole numbers can be split into square numbers.’



**Content Tip:** All natural numbers can be split using square numbers. You let the learners try them from the number 1. You can also give them the four, fours game. Where they use 4, 4, 4, 4 to get 1, 2, 3 and so on. If you let them go up to 10, the first group to finish wins the game

$$1 = (4 + 4) \div (4 + 4)$$

$$2 = (4 \div 4) + (4 \div 4)$$

$3 = (\sqrt{4} + \sqrt{4}) - (4 \div 4)$  and so on



**Skills to be developed:** Analytical thinking and problem solving

### Activity 3.4: Finding Area

#### Suggested Teaching and Learning Materials to Set up a Learning Environment

- **Artificial Environment:** Flip chart, chalkboard/white boards

**What to do:** Provide learners with a rectangular shape.

**Task:** Ask them to calculate the area of the rectangle with length  $\sqrt{3}$  cm and width  $\sqrt{2}$  cm



**Content Tip:** Ask them how they can find the area of this rectangle, Can it be done as  $\sqrt{3} \times \sqrt{2}$  just like  $3 \times 2 = 6$ , then  $\sqrt{3} \times \sqrt{2} = \sqrt{6}$  ask them if we can calculate numbers having  $\sqrt{\quad}$  in the same way. Let them check using numbers such as  $\sqrt{9} \times \sqrt{4}$ . when the conclusion is made let them evaluate more of numbers such as  $\sqrt{18} \times \sqrt{2}$ ,  $\sqrt{7} \div \sqrt{16}$



**Skills to be developed:** Problem solving and analytical thinking

### Activity 3.3: Cube Roots and Real Life Applications

#### Suggested teaching and learning materials to set up a learning environment

- **Artificial environment:** A cubic box, squared paper for each learner, ruler, scissors.

**What to do:** Come up with properly cut squared papers and give each learner.



**Task:** Ask each learner to make a cubic box using a squared paper. Let them place the formed cubic boxes in the bigger box. Ask them to count the number of formed boxes which fill up the bigger box. Let them relate the number of cubic boxes counted to the volume of the bigger box.



**Skills to be developed:** Creativity and Innovation

**Conceptualisation:** Ensure the concept of squares and cubes roots is coming out accordingly. Clearly demonstrate how to evaluate a mixture of square and cube roots and give brief notes.

**Synthesis:** Connect the use of square and cube roots to real life situations

**Evaluation:** Let learners state the square roots of numbers such as 1, 0.01, 0.36, 0.09. Ask them which number is greater than the other e.g.  $\sqrt{7}$  and  $\sqrt{8}$ ,  $-\sqrt{3}$  and  $-\sqrt{2}$ , 4 and  $\sqrt{15}$  they can relate the size of square roots using inequality signs. If they relate 4 to  $\sqrt{16}$  then,  $4 > \sqrt{15}$  Check their understanding by giving real life scenarios on square and cube roots.



**Assessment:** Give learners exercises on evaluating squares and cube roots. Use materials such as: Progress in Mathematics Learners Book Grade 9 page 1 – 10. Zambia Secondary school syllabus Learners Book 10 Page 229.



**Learning Activity 4: Creating Number Patterns and Puzzles**

**Introduction:** Number patterns involve sequences based on specific rules. Identifying and creating patterns enhances problem-solving skills.

**Problem Posing:** *Here is a sequence, 1, 4, 9, 16, 25, \_\_\_\_, \_\_\_\_. What are the next two numbers, and what pattern explains this sequence? Can you create another sequence with different rule?*

### Activity 4.1: Creating Number Patterns

#### Suggested Teaching and Learning Materials to Set up a Learning Environment

- **Artificial Environment:** Flip charts, worksheets.

**What to do:** Put learners in small manageable groups. Ask learners to create a sequence or pattern using square numbers, cube numbers, or rational numbers.

**Task:** Let learners present their patterns to the class. Let the other class members identify the rule in the pattern explaining the logic behind them.



**Content Tip:** When they are done with their presentations let each group draw a table of numbers 1 - 100. Let them identify the 5th multiple of 3, the 9th multiple of 3, the 12th multiple of 3, the 20th multiple of 3, the 24th multiple of 3 and the 100th multiple of 3. The first group to get all answers correct wins.



**Skills to be developed:** Creativity, problem-solving, logical reasoning.

### Activity 4.2: Creating Number Patterns

#### Suggested Teaching and Learning Materials to Set up a Learning Environment

- **Technological Environment:** Simulations, spreadsheet, Desmos, GeoGebra

**What to do:** Put learners in small manageable groups. Ask learners to create a sequence or pattern using technology such as simulations, spreadsheet, Desmos, GeoGebra.

**Task:** Use a spreadsheet or Desmos or GeoGebra to generate number sequences.



**Skills to be developed:** Creativity, problem-solving, logical reasoning.

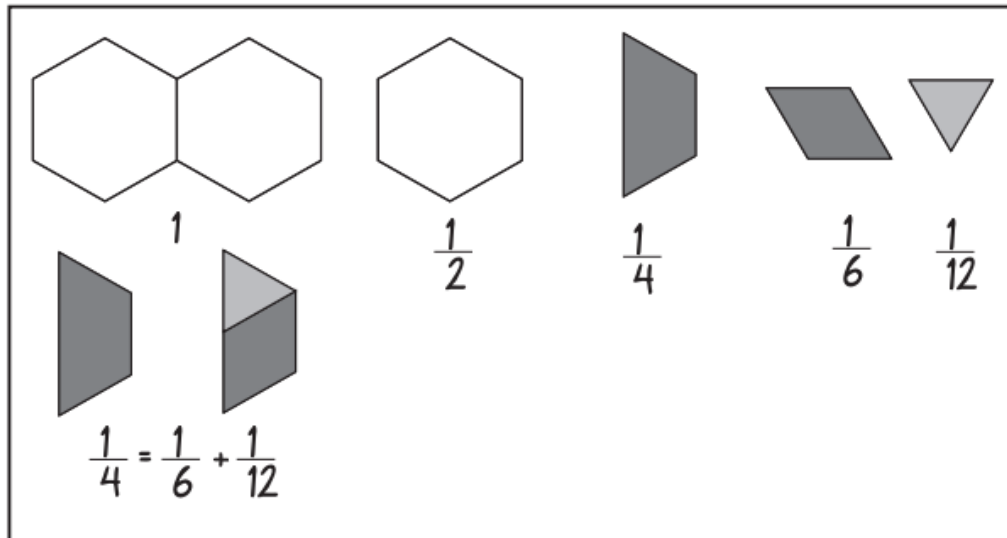
### **Activity 4.3: Pattern Block Puzzles**

#### **Suggested Teaching and Learning Materials to Set up a Learning Environment**

- **Artificial environment:** pattern blocks, cubes, number cards, beads, bottle tops

**What to do:** Divide learners into small groups. Provide learners with pattern blocks, beads, number cards, or cubes to physically create and extend patterns. Set up hands on stations where learners rotate between different types of pattern puzzles (e.g. colour patterns, numerical sequences and shapes).

**Task:** Arrange the objects in a pattern and challenge peers to identify the rule. Predict the next item in a given pattern using logical



reasoning.

				28
				30
				18
				20
?	30	23	22	

$$\begin{aligned} \text{Apple} + \text{Apple} + \text{Apple} &= 18 \\ \text{Apple} + \text{Banana} + \text{Banana} &= 14 \\ \text{Banana} - \text{Cherry} &= 2 \\ \text{Cherry} + \text{Apple} + \text{Banana} &= ? \end{aligned}$$



**Skills to be developed:** Critical thinking, spatial reasoning skills, problem-solving, logical reasoning.

**Conceptualisation:** Make sure the concept of number patterns is coming out accordingly. Clearly demonstrate the difference between rational and irrational numbers (e.g. terminating, recurring) and give brief notes on them.

**Synthesis:** Connect the use of number patterns to real life situations

**Evaluation:** Use a checklist to check the understanding of real-life scenarios on number patterns numbers.



**Assessment:** Give learners a project based assignment to create number patterns using digital tools.

**Expected Standards:** By the end of the learning activities under this topic, learners are expected to use different types of numbers correctly in real life.

**Summative Assessment Guide: Prepare tests, quiz on the following**

- Identify different types of numbers.
- Distinguish between rational and irrational numbers.
- Calculate square and cube roots.
- Create and solve number patterns or puzzles.
- Conduct a quiz with mixed questions on all subtopics.

**Summary:**

**Key Points Recap:**

- Real numbers include rational and irrational numbers.
- Rational numbers can be expressed as fractions, while irrational numbers cannot.
- Square and cube roots are fundamental operations in mathematics.
- Number patterns reveal relationships between numbers and enhance problem-solving.
- Ask open-ended questions to confirm learners' understanding (e.g., “What makes a number irrational?” or “How can you use square roots in real-life situations?”).

## TOPIC 2: SETS

### Introduction

**Overview:** The origins and recognition of sets in Mathematics can be traced back to the late 19<sup>th</sup> century through the works of Georg Cantor (1845-1918), a German Mathematician. Cantor developed the theory of sets and concepts such as cardinality and the distinction between different sizes of infinity. He published his first article about set theory in 1874. It is worth noting that Cantor's work built on earlier concepts of collections and groups of objects but it was his rigorous formalization that laid the ground work for modern set theory. Sets follow some fundamental properties such as Commutative, Associative and Distributive referred to as CAD laws. Sets play a crucial role in various aspects of real-life as they provide a fundamental way of organizing and categorizing objects, events or ideas. Among the real-life aspects where sets are pivotal include organization and classifications of items, database management, social network and relationships, business and marketing as well as sports and competitions.

In this topic, the sub-topics are set builder notations and set operations.

### General Competences:

- **Analytical thinking:** Mastery of set operations and properties.
- **Communication:** Explain clearly and correctly set operations, definitions and notations.
- **Critical thinking:** Identify patterns and solve problems by applying logical, reasoning and set operations.
- **Problem Solving:** Develop skills to solve problems involving sets, including word problems and applications in real-life contexts.



### 2. Key Terms /Words/Vocabulary

- **Cardinality of a set:** The number of elements in a set, e.g.  $n(A)$
- **Complement of a set:** The set of all elements in the universal set that are not in a given set, denoted by say  $A'$
- **Disjoint sets:** Sets that have no element in common. For example if sets A and B are disjoint, then  $A \cap B = \emptyset$
- **Element (member):** An individual object or item in a set. For example  $d$  is an element of the set  $\{a, b, c, d\}$
- **Empty set (null set):** A set without elements, and it is denoted by  $\emptyset$  or  $\{\}$
- **Equal sets:** These are sets that contain exactly the same elements. For example, if sets P and Q are equal, then every element in set P must be in Q and every element in set Q must be in P. This is denoted as  $P = Q$

- **Equivalent sets:** These are sets that have the same number of elements but the elements maybe different. In other words, sets say A and B are said to be equivalent if the cardinality is the same in both sets. This is denoted as  $|A| = |B|$ , but A and B may not contain the same elements
- **Finite set:** This is a set that contains a limited number of elements. In other words, the number of elements in a finite set can be completely counted or listed
- **Infinite set:** This is a set that contains an unlimited or unaccountably large number of elements. In other words, the number of elements in an infinite set cannot be completely counted or listed
- **Improper subset:** A set say R is said to be an improper subset of set S if all elements of set R are in set S, and set R can either be equal to set S or a proper subset of set S. Thus, there is a possibility of  $R = S$ , entailing that every set is an improper subset of itself and an empty set is an improper subset of every set. Hence, every set has only two improper subsets except an empty set. An improper subset is denoted as denoted as  $\subseteq$
- **Intersection Set:** The set of elements common to both sets, denoted by  $A \cap B$
- **Proper subset:** A set say J is said to be a proper subset of say set K if and only if all elements of J are in K but  $J \neq K$ . In other words, J has fewer elements than K, denoted as  $\subset$
- **Roster Method (also known as tabular method):** This is a way to describe a set by listing all of its elements. Each element is written explicitly within curly brackets. It is usually used when a set is small or when the elements are easily identifiable
- **Set:** This is a collection of well-defined distinct objects or elements which can be anything such as numbers or letters. The elements of a set are often listed inside curly brackets, with a comma separating them
- **Set -builder Notation:** This is a concise way to describe a set by specifying the properties that its members must satisfy. It is typically written in the form  $\{x | \text{property of } x\}$  where  $x$  represents an element of the set, the vertical bar can read as “such that” and the property is a condition that elements of the set must satisfy
- **Subset:** A set where all elements are also in another set. For example  $\{e, f\}$  is a subset of  $\{d, e, f, g\}$
- **Union Set:** This is the set of elements in either of the sets given without repeating the elements. It is denoted by the symbol  $\cup$
- **Universal Set:** This is a set containing all possible elements relevant to a particular discussion. It is the “mother” set of all the sets under discussion. It is denoted as  $\mathcal{E}$  = entirety or  $\mathcal{U}$  = universal

- **Venn Diagram:** This is a diagram that uses the rectangle to denote the universal set and circles to represent sets and their relationships

## Sub-Topic 1: Introduction to sets

### Introduction

Sets play a crucial role in various aspects of real life as they provide a fundamental way of organizing and categorizing objects, events or ideas. Thus, exploring sets around us is essential for real life applications. You can introduce the topic sets through a mixture of visual and digital tools, hands-on activities, and real-life examples and scenarios. Doing so will enable learners to develop a strong and intuitive grasp of this essential mathematical concept. The following are some of the learning activities to help learners explore and understand sets. These activities are designed to be engaging, hands-on and relevant to real-life applications.

**Specific Competence:** Apply the concept of set-builder notation and set operations in real life.



### Learning activity 1: Exploring sets in the environment

#### Introduction

Sets are fundamental in Mathematics and help us classify objects, ideas and concepts in a structured way. Set theory connects with the environment, foster observations, analysis, and reasoning skills among learners.



**Problem Posing:** *A community has different sources of water, including lakes, rivers, and wells. Some of these sources are polluted due to human activities.*

- a) *Define a universal set  $W$  representing all water sources and subsets  $C$  (clean water sources) and  $P$  (polluted water sources).*
- b) *How would you use set operations to describe water sources that are both clean and polluted?*
- c) *How can environmentalists use set theory to address water polluted issues?*

### Activity 1.1: Research on Sets

#### Suggested Teaching And Learning Materials to Set up Learning Environment

- **Natural environment:** Daily routine activities

**What to do:** Put learners in small manageable groups of different abilities.

**Task:** Ask learners think about things or items they encounter in daily life that can be grouped together. Let the learners in their findings discover that the mathematical term used for items or things of common characteristics is called a set. After some time, ask some groups of learners make presentations of their findings. Let the other learners in other groups ask questions as one group makes a presentation.

Content Tip: Let learners relate sets to their daily activities



**Skills to be developed:** Research, presentation and teamwork.

### Activity 1.2: Sorting and Classifying Objects

## Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial environment:** Number cards
- **Technological environment:** Digital tools like computers, tablets, smart phones

**What to do:** Put learners into small manageable groups of mixed abilities. Give them number cards to sort out according to type.

**Task:** Ask learners to group number cards based on type such prime numbers, composite numbers, rational numbers. They can also play sorting games of numbers using digital gadgets.



**Skills to be developed:** Collaboration and ICT.

**Conceptualization:** Ensure that the concept of set identification comes out clearly. Give examples of sets from the immediate environment.

**Synthesis:** Connect the use of sets in real-life situations.

**Evaluation:** Use a checklist to check if the learners have understood the concept of sets through practical sorting activities.



**Assessment:** Give learners a project based assignment on how sets are used in the daily operations of businesses and companies entities.



### Learning activity 2: Exploring types of sets

#### Introduction

Through explorations and inquiry based learning, learners will be expected to identify and define different types of sets, and develop critical thinking by analyzing set related problems in diverse scenarios.

**Problem Posing:** *How would the set of stars in the sky be classified and why?*

### Activity 2.1: Research on Sets

#### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial environment:** Books
- **Technological environment:** Digital tools like computers, tablets and smart phones

**What to do:** Put learners in small manageable groups of mixed abilities.

**Task:** Ask learners to explore different types of sets such as infinite, finite sets and make presentations.

**Skills to be developed:** Collaboration, presentation and research.



### Activity 2.2: Empty set/ null set and singleton set

#### Suggested teaching and learning materials to set up learning environment

- **Artificial environment:** Empty chalk box and a chalk box with piece of chalk.

**What to do:** Show an empty chalk box to the learners after which show them a chalk box with a piece of chalk in it. Put learners in small manageable groups of mixed abilities.

**Task:** Let learners discuss real-life scenarios that bring out the concept of an empty and singleton sets in their assigned groups, and present their findings.



**Skills to be developed:** Analytical thinking, collaboration and communication.

### Activity 2.3: Finite and infinite sets

#### Suggested teaching and learning materials to set up learning environment

- **Natural environment:** Sand
- **Artificial environment:** Sugar, mealie meal, pens, sweets, empty box

**What to do:** Put learners in small manageable groups. Give them an empty box which has pens or sweets and mealie meal or sand.

**Task:** Ask learners to count the objects in the given box.



**Content Tip:** After counting, ensure that learners understand that a set with countable or listed objects is known as a finite set and the opposite is an infinite set.



**Skills to be developed:** Collaboration, communication and critical thinking.

#### Activity 2.4: Intersection and Union Set

##### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial environment:** Work sheets

**What to do:** Put learners in small manageable groups with mixed abilities.

**Task (a):** Ask each learner in their small groups to list his or her favorite sport (s) in Grade 7. Let them compare their lists in their small groups and write the common favorite sports. Have learners come up with a mathematical term in the context of sets which represents common elements of two or more sets including its symbol.

**Task (b):** Ask learners to combine all the favorite sports in **task (a)** into one list. Let them discuss the mathematical term used to describe the set which they have formed including its symbol.

**Skills Developed:** Analytical thinking, collaboration and communication.

**Conceptualization:** Ensure that the concept of types of sets comes out clearly. Explain clearly on the following types of sets and give brief notes on them: Empty/null, singleton, finite, infinite, equal, equivalent, complement, intersection and union sets.

**Synthesis:** Connect the use of types of sets in real-life situation.

**Evaluation:** Check that learners have the basic understanding and quick recall of concepts related to different types of sets.

**Assessment:** Give individual exercise to the learners from the Zambia Secondary School Syllabus Mathematics Learners Book 10 (pages 25 to 30).



### Learning Activity 3: Interpreting Set-builder Notation

#### Introduction

Set-builder notation is very essential as it helps learners to express sets more precisely and compactly especially when describing large or infinite sets. It also allows for a clear and efficient way to define sets using a rule or condition, rather than just listing elements individually. Doing so fosters logical thinking and deeper understanding of how mathematical structures can be represented and manipulated. For more information on set-builder notation, visit <https://study.com/academy/lesson/how-to-write-sets-using-set-builder-notation.html#/lesson>.

**Problem Posing:** Let  $A = \{x|x \text{ is an integer and } 1 \leq x \leq 5\}$ . What are the elements of set A?

#### Activity 3.1: Research on Set Presentation

##### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial environment:** Books
- **Technological environment:** Digital tools like computers, tablets and smart phones

**What to do:** Put learners in small manageable groups of mixed abilities.

**Task:** Ask learners to explore different ways of presenting sets and make presentations.



**Content Tip:** Interpret set builder notation such as,  $\{x|x \in y\}$  or  $\{x:x \in y\}$ .



**Skills to be developed:** Collaboration, presentation, research and ICT.

### Activity 3.2 Video Showing

#### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Technological environment:** Digital tools like computers, overhead projector, tablets and smart phones

**What to do:** Download different types of videos on set-builder notation. Put learners in small manageable groups of mixed abilities. Explain to the learners why you want to show the videos and what learners should pay attention to.

**Task:** Let learners watch the videos attentively and discuss the different ways of presenting sets. Thereafter, let learners in their respective groups come up with different sets presented in different ways. Let each group present their findings to other class members.



**Skills to be developed:** Collaboration, communication and ICT skills.



#### Learning activity 4: Presenting Sets in Set-builder Notation

Set builder notation is one of the ways of representing sets in the most convenient and concise way.

#### Activity 4.1: Sorting Numbers by Type

#### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial environment:** Number cards (natural, whole, prime, integers, even, rational and irrational).

**What to do:** Put learners in small manageable groups of mixed abilities. Give learners number cards.

**Task:** Ask learners to sort the number cards based on their type. Let the learners express the set of numbers that meet the certain criteria using set builder notation. For example, “the set of prime numbers” can be written as  $\{x|x \text{ is a prime number}\}$ .



**Content Tips:** Let learners come up with different set-builder notations for natural, whole, prime, integers, even, rational and irrational numbers.



**Skills to be developed:** Collaboration, communication and critical thinking

#### Activity 4.2: Sports Team or Club Membership

## Suggested Teaching and Learning Materials to Set up Learning Environment

### ➤ **Natural Environment:** Learners

**What to do:** Organize a game or activity that involves learners joining teams or clubs based on specific criteria, such as age or experience.

**Task:** Ask learners to describe the set of members of a team or club using set builder notation. For example, “the set of all members in the soccer club who are over 10 years old” can be written as  $\{x|x \text{ is a member of the soccer club and } x > 10\}$ .



**Skills to be developed:** Team work and communication.

**Conceptualization:** Ensure that the concept of presenting sets using set-builder notation clearly comes out and give brief notes.

**Synthesis:** Connect the use of set-builder notation in real-life situation.

**Evaluation:** Check that learners have the understanding of the concept of set-builder notation through open-ended question.



**Assessment:** Give exercises on set builder notation from Learners textbooks such as “Progress in Mathematics Learners Book 8 on page 3 or Excel and Advance in Mathematics Learner’s Book Grade 8 on pages 2 and 3.” Give project based assessments to evaluate long term understanding and application of set-builder notation. For example, have learners create a project (categorize a collection of objects, survey data or media items) and present their findings, expressing their results using set builder notation.

## Sub-Topic 2: Operations on Sets

### Introduction

Operations on sets are fundamental tools in Mathematics used to combine, manipulate and compare sets. These operations help in organizing data and solving problems across various fields that include computer science, statistics and everyday life. The main set operations are intersection, union and complement. These operations can be combined and this brings out the concept of combined operations. Understanding these operations is essential for logical reasoning and data analysis. For deeper understanding of operations on sets by the learners, it is encouraged that visual aids, digital platforms and learning activities based on real-life applications are used. In this subtopic, the operations on sets should go up to three sets.

**Problem Posing:** *A community organizes a tree planting event where 20 learners' plant fruit trees, 15 learners' plant shade trees and 10 learners plant both fruit and shade trees.*

- a) *How many learners participated in at least one type of tree planting?*
- b) *How many learners planted only fruit trees?*
- c) *How many learners planted only shade trees?*
- d) *If the class has 40 learners, how many did not participate in the tree planting?*



### **Learning Activity 5: Performing Single and Combined Operations Involving Intersection, Union and Complement of Sets**

**(combining up to three sets)**

#### **Activity 5.1: Intersection and Union Set**

#### **Suggested Teaching and Learning Materials to Set up Learning Environment**

- **Artificial Environment:** Work sheets

**What to do:** Put learners in small manageable groups with mixed abilities.

**Task (a):** Ask learners in respective groups list their favorite fruits. Let them compare their lists in their respective groups and write the three common favorite fruits. Have learners come up with a mathematical term in the context of sets which represents common elements of three sets including its symbol.

**Task (b):** Ask learners to combine all the three favorite fruits from task (a) into one list. Let them discuss the mathematical term used to describe the set which they have formed including its symbol.





**Skills to be developed:** Analytical thinking, collaboration and communication.

### Activity 5.2: Universal and Complement of a Set

#### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Natural environment:** Fruits

**What to do:** Put learners in small manageable groups with mixed abilities as in **activity 5.1**.

**Task:** (i) Ask learners in their groups list all their favorite fruits.

(ii) Let learners describe the set to which their common favorite fruits belong using a mathematical term.

(iii) Ask learners list the set of fruits described in (ii) above.

(iv) Let the learners list the set of fruits which are not in the set listed in (iii) above.

(v) Ask learners to state the mathematical name given to the set listed in (iv) above.

(vi) Let learners give a symbol that is used to describe the set.



**Content Tip:** For part (ii), the term which should come out is universal set. Then for part (iv), the term that should be complement. For part (vi), the symbol should be  $A'$  or  $A^c$ .



**Skills to be developed:** Collaboration and communication.

### Activity 5.3: Combined Operations on Sets

#### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial environment:** Letter cards

**What to do:** Put learners in small manageable groups with mixed abilities.

**Task:**

- (i) Ask the learners in their respective groups list all the letters of the English alphabet. Let them denote this set as set E.
- (ii) Let learners in their respective groups list the set of vowels in E and denote it as set A.
- (iii) Ask learners in their respective groups to list letters from letter *c* to letter *n* from set E. Let them name this set as set B.
- (iv) Let learners in their respective groups list the letters of the word MATHEMATICS. Let learners name this set as set C.
- (v) Ask learners in their respective groups list the set of letters found in both set A and C.
- (vi) Let learners in their respective groups list the set of letters in set E that are not common to sets A and C.
- (vii) Ask learners in their respective groups to list the set of letters that are not in set A but belong to both set B and C.
- (viii) Let learners in their respective groups list the set of letters that do not belong to set B or set C but belong to set A.



**Content Tip:** Explore further combinations of different sets for learners' examples to deepen their understanding.



**Skills to be developed:** Analytical thinking, collaboration, communication, critical thinking and problem solving.

**Conceptualization:** Ensure that the concept of operations on sets clearly comes out. Make sure that brief notes on operations on sets are given.

**Synthesis:** Connect the use of operations on sets in real-life situation.

**Evaluation:** Check that learners have deeper understanding of operations on sets in real-life.



**Assessment:** Give individual, pair work or group work questions on set operations using learners' textbooks such as "Progress in Mathematics Learners Book 8" learner activity 2, 3, 4 and 5 in on page 5, 7, 9 and 10 respectively, Exercise 1.2, 1.3, 1.4 and 1.5 in "Excel and Advance in Mathematics Learner's Book Grade 8" on pages 5, 7, 9 and 13 respectively, exercise 2a in "Secondary School Mathematics: A course for Grades 10-12" on pages 8&9, activity 2 in "Progress in Mathematics Learners Book 10" on page 6. Give Project Based Assessments.



**Learning Activity 6: Illustrating Sets Using Venn Diagrams**

## Introduction

The origins of the Venn diagram as a mathematical concept can be traced to the works of John Venn (1834-1923) who was an English Mathematician, Philosopher and Logician. The Venn diagram was designed to illustrate the logical relationships between different sets. The goal of John Venn was to create a simple way of visually representing set theory which was developing as a mathematics area at that particular time. In the Venn diagram, the overlapping circles show the interaction of sets (whether they share common elements, have unique elements or are entirely distinct). The geometrically rectangular shape in the Venn diagram indicates the universal set while the circles represent the subsets of that universal set. Connecting Venn diagram to familiar and everyday situation allows learners to see its usefulness and develop a deeper understanding of how mathematical concepts apply to real life. The use of everyday situations in Venn diagrams promotes higher order thinking.

**Problem Posing:** *In a school sports completion, learners had two favorite sports. 50 learners play football (F), 30 learners play basketball (B) and 15 learners play both football and basketball. I tried to count all the football and basketball players together, and I got 80. But I made a mistake because some learners were counted twice. How many learners actually play at least one sport?*

### Activity 6.1: Video showing

#### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Technological environment:** Digital tools like computers, overhead projector, tablets and smart phones

**What to do:** Download different types of videos on Venn diagrams. Put learners in small manageable groups of mixed abilities. Explain to the learners why they should watch the videos and what they should pay attention to. Give each group a question which they should demonstrate after watching the videos.

**Task:** Let learners watch the videos attentively and observe how information is illustrated in the Venn diagram. Thereafter, let learners in their respective groups demonstrate how to illustrate given information in the Venn diagram. Let each group present their findings to other class members.



**Content Tip:** Give learners questions which bring out the illustration of the concept of Venn diagram.



**Skills to be developed:** Collaboration, communication and ICT skills.

**Conceptualization:** Ensure that the concept of Venn diagram is clearly explained. Brief notes on Venn diagrams should be given.

**Synthesis:** Connect the use of Venn diagrams to real-life situation.

**Evaluation:** Check that learners have deeper understanding of Venn diagrams in real-life applications through open-ended questions.



**Assessment:** Give learners real-world problem solving tasks. For example, present a real-world scenario and ask learners to create a Venn diagram to solve problems. Learners can also be assessed using project based assessments. Give learners exercises from the Zambia Secondary School Mathematics by Finch (pages 10 to 19).



### **Learning Activity 7: Finding the Number of Elements in a Given Set, $n(A)$**

#### **Introduction**

The concept of the number of elements in a set is fundamental in Mathematics and is referred to as the cardinality of the set. The cardinality represents how many distinct elements are in the set. This concept has several practical applications in real life such as in counting, organizing data or objects. For example, the concept of cardinality is of much help in inventory management. To illustrate this, a store owner may want to track the number of distinct items in stock. If the store sells shirts, pants and shoes, the set of items in stock can be represented and the number of elements (cardinality) in the set indicates how many unique types of products are available. Another vital real-life application of the cardinality of sets is classroom attendance. In this case, the number of learners present in a classroom at

any given time can be viewed as the cardinality of a set, where each learner represents unique elements. This ultimately helps in tracking attendance or participation in class activities.

**Problem Posing:** *In a group of 50 people, 25 speak English, 20 speak French, and 10 speak both. How many people speak at least one of the two languages?*

### Activity 7. 1: Sorting by Colour

#### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial Environment:** Bottle tops or marbles of different colors.

**What to do:** Put learners into small manageable groups of mixed abilities. Give learners bottle tops or marbles of different colors.

**Task:** Ask the learners in their respective groups form sets based on different criteria (for example set A: red bottle tops or marbles, set B: blue bottle tops or marbles and set C: white bottle tops or marbles). Then let learners count how many elements are in each set and present them in set notation form,  $n(A)$ .



**Skills to be developed:** Collaboration, communication and problem solving.

**Conceptualization:** Make sure that the concept of number of elements in a given set is clearly explained.

**Synthesis:** Connect the use of number of elements in a given set to real-life situation.

**Evaluation:** Check that learners have an understanding of number of elements in a given set to real-life applications by asking open-ended questions.



**Assessment:** Give individual learners real life scenario word problems and then ask learners to find the number of elements in the set. Assess learners through portfolio activities.



## Learning activity 8: Describing the Shaded Region Using Set Notation and Shading Regions Described by a Set Notation

### Introduction

In Mathematics, shading of regions in a Venn diagram is used to represent sets, conditions and relationships between various quantities or objects.

**Problem Posing:** *In a Venn diagram with three sets  $A$ ,  $B$ , and  $C$ , the regions where  $A$  overlaps with  $B$ ,  $B$  overlaps with  $C$ , and  $A$  overlaps with  $C$  are all shaded, but the region where all three sets overlap is left unshaded. What does the shaded region represent? Draw the Venn diagram and shade the correct region.*

### Activity 8. 1: Venn diagram shading

#### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial Environment:** Manila paper/flip chart and markers of different colors.
- **Technological Environment:** Computers, interactive software (like Geogebra or Desmos) and tablets.

**What to do:** Divide the class into small manageable groups of mixed abilities. Provide learners with questions on sets involving Venn diagrams. Provide learners with questions where they can either use manila paper/flip charts or digital gadgets.

**Task:** Ask the learners to shade specific regions based on set operations (union, intersection or complement).



**Content Tip:** Give learners questions up to the combination of three sets.



**Skills Developed:** Collaboration, critical thinking and problem solving.

### Activity 8.2: Interactive Venn diagram software

#### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Technological Environment:** Computers or tablets with access to interactive Venn diagram software (for example Geogebra or Demos).

**What to do:** Divide learners into small groups of mixed abilities. Use an interactive platform where learners can manipulate the shaded regions on a Venn diagram.

**Task:** Let learners manipulate the shaded regions on a Venn diagram. Ask learners to write the corresponding set notation. For example, if the union of sets A and B is shaded, then they would write  $A \cup B$ .



**Skills to be developed:** Collaboration and ICT skills.

### Activity 8.3: Set Notation Puzzle

#### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial Environment:** Pre-made puzzle cards with Venn diagram images and set notations

**What to do:** Provide learners with puzzle cards where one side shows a shaded region of a Venn diagram and the other side shows multiple set notation options.

**Task:** Ask learners to match the correct set notation with the shaded region.



**Skills to be developed:** Collaboration, communication and critical thinking.

### Activity 8.4: Venn diagram shading

#### Suggested Teaching and Learning Materials to Set up Learning Environment

**Artificial Environment:** Paper, markers, Venn diagram templates

**What to do:** Divide the learners into small groups of mixed abilities.

**Task:** Ask some groups to create a Venn diagram based on specific set operations (such as union, intersection or complement). Then, let the other groups write the set notations that correspond to the shaded region.



**Skills to be developed:** Collaboration, communication and critical thinking.

**Conceptualization:** Make sure that the concept of describing the shaded region using set notation and shading regions described by a set notation is clearly explained.

**Synthesis:** Connect the concept of describing the shaded regions using set notation and shading regions described by a set notation to real-life situation.

**Evaluation:** Check that learners have an understanding of describing the shaded regions using set notation and shading regions described by a set notation to real-life applications.



**Assessment:** Give learners projects that bring out the concept of shading regions. Give learners exercises from learners' textbooks such as the Zambia Secondary School Mathematics by Finch (pages 15 - 19).



### **Learning Activity 9: Applying Commutative, Associative and Distributive Laws of Operations on Sets.**

#### **Introduction**

The commutative, associative and distributive laws are fundamental properties that describe how operations on sets behave. These laws provide structure and consistency to the operations of union and intersection in set theory much like how the corresponding properties apply to numbers in arithmetic. Further, these laws help in understanding how sets interact with each other and allow for the manipulation and simplification of set expressions in Mathematics and logic. Therefore, for better understanding of these CAD laws by the learners, it is encouraged that real-life scenarios from the immediate environment of the learners are used during lessons.



**Problem Posing:** *I am a collection of elements, and you can combine me with another collection. When you switch the order of collections, I don't change. If you group them differently, I still stay the same. When I distribute over another operation, the result is the same as doing the operations first, then combining. Who am I?*

### Activity 9.1 Commutative Law on Sets

#### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial Environment:** Exercise books and pens

**What to do:** Divide the class into small manageable groups with mixed abilities.

#### Task:

- Ask the learners in their respective groups' to list whole numbers from 0 to 10 and denote this set as set A.
- Let learners in their respective groups list the set of natural numbers from 0 to 10 and denote it as set B.
- Ask learners in their respective groups to list even numbers from 0 to 10 and denote it as set C.
- Ask learners in their respective groups to find  $A \cap B \cap C$  and  $C \cap B \cap A$ . Let them make observations.
- Let learners in their respective groups to find  $A \cap (B \cap C)$  and  $A \cap (B \cap C)$ . Let them make observations.
- Ask learners in their respective groups to find  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ . Let them make observations.



**Content tip:** Ensure that all properties of CAD laws are clearly explored.



**Skills to be developed:** Analytical thinking, collaboration, communication, critical thinking and observation.

**Conceptualization:** Ensure that the concepts of commutative, associative and distributive laws referred to as CAD laws are clearly explained. Give concise notes on these laws for deeper understanding.

**Synthesis:** Connect the concepts of commutative, associative and distributive laws to real-life situation.

**Evaluation:** Present real-life scenarios where learners must apply the CAD laws.



**Assessment:** Providing them with incomplete equations or statements that learners should fill in.

**Expected Standards:** By the end of this topic the concept of set builder notation and set operations should be applied correctly.

**Summative Assessment:** Carry out a survey to find out your classmates' interests in the three categories of sports hobbies and favourite music. Record the data in a table. Create well defined sets for the three categories and perform set operations. Analysis and present the data to the class.



**Summary:**

**Key Points Recap:**

- There are so many objects that conform to the concept of set within our environment
- The different types of sets include empty/null, singleton, equal, equivalent, intersection, complement and union sets
- Sets can be presented in different ways such as Roster/Tabular/Listing and set-builder notation
- Set-builder notation helps to express sets more precisely and compactly especially when describing large or infinite sets
- Intersection, union and complement sets are the set operations that can be applied either on single basis or combined
- Venn diagrams are a diagrammatical way of illustrating the given set information.
- Commutative, associative and distributive laws are fundamental properties that describe how operations on sets behave
- The use of Information and Communication Technologies is very essential for deeper understanding of the concept of sets.

## TOPIC 3: INTEGERS

### 1. Introduction

**Overview:** Integers are the fundamental building block of Mathematics, representing whole numbers that are either positive, negative or zero. The word “integer” originates from the Latin word meaning “whole” or “untouched” reflecting their role as complete and indivisible units. This means that integers are numbers that can be written without a fractional part, such as 3, 6, 7, -3 and -8. All decimal numbers like  $\frac{2}{3}$ , 4.75 or  $\sqrt{5}$  are not considered as integers. The set of integers is denoted by a symbol  $\mathbb{Z}$  ( $Z$ ) from a Germany word “Zahlen”, meaning “numbers”. This symbol  $\mathbb{Z}$  ( $Z$ ) stands for the set of all integers such that,  $\mathbb{Z}$  ( $Z$ ) =  $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ . Integers play a crucial role in various mathematical operations such as addition, subtraction, multiplication and division. They follow five fundamental properties which are closure, associative, distributive, commutative, and identity. These properties are essential for various algebraic calculations in solving equations and inequalities, arithmetic and number theory. The four operations is the sub-topic of integers.

#### General Competences:

- **Analytical Thinking:** Evaluate solution

- **Communication:** Use appropriate mathematical language in different situations
- **Creativity and Innovation:** Create puzzles and patterns using different types of numbers
- **Critical Thinking:** Solve complex problems
- **Digital Literacy:** To upload and download information
- **Problem Solving:** Present reasoned explanations for phenomena



## 2. Key Terms/Words/Vocabulary

- **Integers:** These are whole numbers either positive, negative or zero without a fractional part and are denoted as  $\mathbb{Z}$
- **Positive integers:** These are integers greater than zero and are denoted as  $\mathbb{Z}^+$
- **Negative integers:** These are integers less than zero and are denoted as  $\mathbb{Z}^-$
- **Zero:** This is a unique integer that represents neutrality, balance or starting point. It is neither negative nor positive and it separates positive and negative integers

### Sub-Topic: The Four Operations on Integers.

#### Introduction

Performing arithmetic operations on integers involves applying specific rules and procedures to obtain accurate results. The four basic operations; addition, subtraction, multiplication and division can be applied to integers to solve mathematical problems and real-world applications.

**Specific Competence:** Apply the four operations on integers in real-life situations.



#### Learning Activity 1: Exploring Integers

**Introduction:** Exploring integers starts with understanding their basic properties and how they interact in different mathematical situations and operations. They are commonly used in everyday life from counting to measuring and keeping track of financial transactions. Integers can be introduced through a mixture of visual tools, hands-on activities and real-life examples that can make learners develop a strong and intuitive grasp of essential mathematical concepts. Here are several engaging activities to help learners explore and understand integers.

**Problem Posing:** *I am an integer. My absolute value is 6. I am greater than  $-10$  but less than  $0$ . What number am I?*

### **Activity 1. 1: Research on Integers**

#### **Suggested Teaching and Learning Materials to Set up Learning Environment**

- **Artificial Environment:** Books
- **Natural Environment:** Objects in the environment
- **Technological Environment:** Computers, tablets, iPad, calculators and phones

**What to do:** Put the learners in manageable groups of mixed abilities.

**Task:** Ask the learners to conduct some research on integers and make presentations to the class. Let the other learners ask questions and choose the best presentation to be winners.



**Skills to be developed:** ICT skills, presentation, research and team work.

### **Activity 1. 2: The Zero Bank Balance**

#### **Suggested Teaching and Learning Materials to Set up Learning Environment**

- **Artificial environment:** Paper, Pen and Money

**What to do:** Guide the learners on how people deposit and withdraw money at the bank and put the learners in manageable groups of mixed abilities to act a scenario at the bank. Give each group an amount of money to deposit (K50.00) and allow them also to withdraw (K20.00). Create more situations where their withdrawals equal their deposits which bring the balance to zero. Ask them to explain what

zero means. Give more follow-up questions (e.g. What happens when you withdraw more money than you have? Let learners use terms such as positive balance, moving away from zero, going below zero, negative balance, no value, neutral).

**Task:** Let learners discuss how they are going to write in their books the deposits and withdraws. Let them explain the meaning of the transactions made the class.



**Skills to be developed:** Collaboration, communication, financial literacy and problem solving.

**Conceptualization:** Ensure that the concept of integers is coming out correctly.

**Synthesis:** Connect integers to real-life situation.

**Evaluation:** Check if learners can work with integers in different situations.



**Assessment:** Let learners investigate how integers are used in real-life.



## **Learning Activity 2: Comparing situations which depict integers**

### **Introduction**

Integers are everywhere even though we might not recognize them. In all reality, integers are just numbers, but a way to represent both positive and negative values that affect our daily lives in meaningful ways. Hence, there is need to guide the learners to come up with scenarios that depict the integers around us through the use of real-life situations. Here are some suggested activities; comparing situations which depict integers such as loss and profit in business, low and high temperatures, elevation below and above sea level.

**Problem Posing:** *In a football league, teams earn 3 points for a win, 0 points for a loss, and 1 point for a draw. Team A has 5 wins, 3 losses, and 2 draws. Team B has 4 wins, 4 losses, and 2 draws.*

- a) *Calculate the total points for each team using integers.*
- b) *Compare the performance of Team A and Team B. Which team has more points?*
- c) *If team A losses 2 more matches and Team B wins 1 more match, what will be their new standings?*

## Activity 2. 1: Negative Numbers in the Environment

### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial Environment:** Paper and thermometer
- **Natural Environment:** Water

**What to do:** Put learners into small manageable groups to come up with the concept of negative numbers and provide them with paper.

**Task:** Ask learners to measure the temperature of water (room temperature water, cold water from the fridge and ice cold water). Let them record and interpret their findings. The group leaders should make presentations to the class.



**Content Tip:** Give learners more activities that bring out the concept of negative integers in real-life.



**Skills to be developed:** Analytical thinking, communication, research and team work.

## Activity 2. 2: Exploring Elevations Above and Below Sea Level

## Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial environment:** Work sheets
- **Technological environment:** Computers and internet

**What to do:** Divide learners in small manageable groups. Provide each group with a work sheet containing data on 5 to 10 Zambian hills, escapements, mountains and falls.

**Task:** Let learners identify whether each location is above or below sea level. Let learners research and note elevation of each location. Ask learners to discuss how the elevation might affect the climate, vegetation, and human activities in each location. Ask the groups to present their findings to the class.

**Content tip:** On the data work sheet, put popular sites such as Muchinga escapement, Victoria falls, Lumangwe falls, Chishimba falls and Mafinga hills.

**Skills to be developed:** Collaboration, critical thinking and ICT skills.

**Conceptualization:** Ensure the learners are bringing out situations that depict integers from the environment.

**Synthesis:** Connecting integers to real-life situation.

**Evaluation:** Check if the learner can work with zero, positive and negative numbers in real life situation.

**Assessments:** Give learners a research topic on how the elevator operates in relation to integers.



### Learning Activity 3: Adding Integers

#### Introduction

Adding integers is a fundamental concept in Mathematics, essential for understanding more complex operations. The process of adding integers involves combining given numbers to find their sum.



**Problem Posing:** *A quiz competition awards +5 points for correct answers and -2 points for incorrect answers. If a learner answers 6 correctly and 3 incorrectly, what is the final score?*



### **Activity 3. 1: Integer Card Game (Addition)**

#### **Suggested Teaching and Learning Materials to Set up Learning Environment**

- **Artificial environment:** Number cards and paper.

**What to do:** Divide learners into small manageable groups. Prepare numbers cards from numbers (-10) up to (+10) and give each group.

**Task:** Let the learners shuffle the cards and share equally. Each learner picks two cards and must find the sum (by adding), the learner with the highest sum wins the game. Let the game continue for 5 to 10 rounds and the learner with the highest wins is the champion.

**Skills to be developed:** Collaboration, communication, critical thinking and problem solving.



### **Activity 3. 2: Integer Number Line Walk**

#### **Suggested Teaching and Learning Materials to Set up Learning Environment**

- **Artificial environment:** A large floor number line, box and work cards

**What to do:** Put the learners in small manageable groups. Let them draw larger number lines on the floor/ground so that they can play the game guided by the following steps,

Neutral position= Always at zero

Positive sign of the number = move forward

Negative sign of the number = move backward

(+) sign(operation) = face positive direction

(-) sign (operation) = face negative direction.

**Task:** Let learner follow guided steps to pick a work card from the box and play the game. Make sure all the members of the group participate.

1. Add (+2) and (+3)

i. Stand at zero facing in the positive direction.

ii. The positive sign of the number means ‘move forward’. Hence, +2 mean that move 2 steps forward.

iii. The operation sign + means face in the positive direction.

iv.+3 means move 3 steps forward.

v.The learner has arrived at +5. Therefore,  $+2 + (+3) = +5$

2.  $+3 + (-5) =$

i. Stand at zero facing in the positive direction.

ii. +3 means move 3 steps forward.

iii. The operation signs, + means face in the positive direction.

iv. The negative sign of a number means moves backward. Hence -5 means move 5 steps backward.

v. The learner has arrived at -2. Therefore,  $+3 + (-5) = -2$ .

**Content Tip:** Give learners as many questions as possible on addition of integers. Ensure that the rules of adding integers are fully explored.

**Skilled to be developed:** Interpersonal, observation and teamwork.

**Conceptualization:** Make sure the learners bring out the concept of adding integers correctly. Explain clearly on integers and give brief notes.

**Synthesis:** Connecting addition of integers to real-life.

**Evaluation:** Check if the learners can identify and work with addition of integers.

**Assessments:** Give learners exercises on adding integers using Learners Textbooks such as Progress in Mathematics, Grade 8 (pages 15 - 16), Zambia Basic Education Course Grade 8 (pages 51 - 56) or any other helpful resource.



#### Learning activity 4: Subtracting integers

## Introduction

The subtraction of integers is a fundamental mathematical operation that involves finding the difference between two numbers.

**Problem Posing:** *A hiker is at 10 meters above sea level. She descends to -2 meters below sea level.*

- a) *What is the total change in her elevation?*
- b) *How can subtraction help you solve this problem?*

### Activity 4. 1: Integer Card Game (Subtraction)

#### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial environment:** Number cards and paper.

**What to do:** Divide learners into small manageable groups. Prepare numbers cards from numbers (-10) up to (+10) and give each group.

**Task:** Let the learners shuffle the cards and share equally. Each learner picks two cards and must find the difference (by subtracting), the learner with the lowest difference wins the game. Let the game continue for 5 to 10 rounds and the learner with the highest wins is the champion.



**Skills to be developed:** Collaboration, communication, critical thinking and problem solving.

### Activity 4. 2: Integer Number Line Walk (Subtraction)

#### Suggested Teaching and Learning materials to set up learning environment

- **Artificial environment:** A large floor number line, box and work cards

**What to do:** Put the learners in small manageable groups. Let them draw larger number lines on the floor/ground so that they can play the game guided by the following steps,

Neutral position= Always at zero

Positive sign of the number = move forward

Negative sign of the number = move backward

(+) sign(operation) = face positive direction

(-) sign (operation) = face negative direction.

**Task:** Let learner follow guided steps to pick a work card from the box and play the game. Make sure all the members of the group participate.

1. Subtract (+4) from (+3)

i. Stand at zero facing in the positive direction.

ii. The positive sign of the number means ‘move forward’. Hence, +3 mean that move 3 steps forward.

iii. The operation sign – means face in the negative direction.

iv.+4 means move 4 steps forward.

v.The learner has arrived at  $-1$ . Therefore,  $+3 - (+4) = -1$

2.  $-3 - (-5) =$

vi. Stand at zero facing in the positive direction.

vii.–3 means move 3 steps backwards.

viii.The operation signs, – means face in the negative direction.

ix.The negative sign of a number means moves backward. Hence -5 means move 5 steps backward.

v.The learner has arrived at  $+2$ . Therefore,  $+3 + (-5) = +2 = 2$



**Content tip:** Give learners as many questions as possible on subtraction of integers. Ensure that the rules of subtracting integers are fully explored.



**Skilled to be developed:** Interpersonal, observation and teamwork.

### Activity 4. 3: Integer Dice Battle

## Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial Environment:** Two dice of different colours, score sheet

**What to do:** Put the learners in small manageable groups. Provide the groups with two dice of different colours (one colour to represent positive number and the other negative numbers).

**Task:** Ask learners in their respective groups to roll the two dice one after the other 5 times. Let them record the result on their score sheets. Ask the learners to subtract the result of the second die from the result of the first die. The group with the highest score after 5 rounds wins the battle.



**Skills to be developed:** Computation fluency and collaboration.

**Conceptualization:** Make sure the learners bring out the concept of subtracting integers correctly.

**Synthesis:** Connecting subtraction of integers to real-life.

**Evaluation:** Check if the learners can identify and work with subtraction of integers.



**Assessments:** Give learners exercises on subtracting integers using Learners Textbooks such as Progress in Mathematics, Grade 8 (page 16), Zambia Basic Education Course Grade 8 (pages 56 - 61) or any other helpful resource.



### Learning Activity 5 – Multiplying Integers

#### Introduction

Mastering multiplication of integers is essential for solving real-life problems such as calculating temperature, financial transactions and mathematical equations. There is need for the learners to understand the rules that govern these operations.

**Problem Posing:** *A penalty in a game deducts  $-3$  points per mistake. After 4 mistakes, what is the total score change?*

*a) What is the result of the multiplication?*

*b) Does the score increase or decrease? Explain why*

### **Activity 5. 1: Integer Dice Battle**

#### **Suggested Teaching and Learning Materials to Set up Learning Environment**

- **Artificial Environment:** Two dice of different colours, score sheet

**What to do:** Put the learners in small manageable groups. Provide the groups with two dice of different colours (one colour to represent positive number and the other negative numbers).

#### **Task:**

- Ask learners in their respective groups to roll the two dice one after the other 5 times. Let them record the result on their score sheets. Ask the learners to multiply the results of the dice for each roll for both colours. The group with the highest score after 5 rounds wins the battle.
- Ask the learners in their respective groups to roll the die of the same colour (positive and negative) twice and record the results on the score sheet (let them do that twice for the same colour). Ask learners to multiply the results of the dice for each roll on the score sheet. The group with the highest score wins the battle.

**Skills to be developed:** Collaboration, communication, computation fluency and observation.

### **Activity 5. 2: Integer Jump Race**

#### **Suggested Teaching and Learning Materials to Set up Learning Environment**

- **Natural Environment:** Number line drawn on the floor or ground

➤ **Artificial environment:** Work sheets

**What to do:** Put the learners in small manageable groups. Give learners worksheets having multiplication questions. The jumper always starts at zero on the number line.

**Task:** Let the learners use the number line to demonstrate the multiplication statements given on the worksheet. Let them record their answers on the worksheet.

**Rules:**

- a) The jumper always stands at zero facing the positive direction,
- b) Positive  $\times$  positive: (e.g.  $+3 \times +4$ ) means the jumper moves three steps forward per round 4 times.
- c) Negative  $\times$  positive: (e.g.  $-2 \times 5$ ) means the jumper moves 2 steps backwards per round 5 times
- d) Negative  $\times$  negative: the jumper realizes that instead of moving backwards, they move forward (e.g.  $-2 \times -3$ ) means the jumper moves 2 steps forward 3 times, the second negative reverses the direction of the first one, hence moving forward.



**Skills to be developed:** Collaboration, communication, computation fluency and observation

**Conceptualization:** Make sure the learners bring out the concept of multiplying integers correctly. Explain clearly on multiplication of integers and give brief notes.

**Synthesis:** Connecting multiplication of integers to real-life situations.

**Evaluation:** Check if the learners can multiply integers.



**Assessments:** Give learners exercises on multiplication of integers and ensure they can apply them to real-life.



**Learning Activity 6: Dividing of Integers**

**Introduction**

In our daily lives, we often share and divide things equally. Division is a process of splitting a number into equal parts. The concept of division will equip learners with the skills of sharing, distributing, comparing and solving problems efficiently.

**Problem Posing:** If  $a$  and  $b$  are integers, and  $\frac{a}{b}$  is not an integer, what can you say about  $a$  and  $b$ ? Justify your answer with an example.

### Activity 6. 1: Exploring Division of Integers

#### Suggested Teaching and Learning Materials to Set up Learning Environment

➤ **Artificial Environment:** Worksheets

**What to do:** Put learners into small manageable groups of mixed abilities. Give each group a work sheet to solve the given questions on division of integers. The work sheets should contain a pattern of questioning such as,

- Both the dividend and the divisor are positive
- The dividend to be positive and the divisor is negative
- The dividend to be negative and the divisor is positive
- Both the dividend and divisor are negative

**Task:** Ask the learners to work out the questions from the work sheets and make presentations. Encourage other learners to ask questions. The best presenters are the winners.



**Skills to be developed:** communication, collaboration and critical thinking.

**Content tip:** Give learners more activities that bring out the sign rules and real-life situations.

**Conceptualization:** Make sure the learners bring out the concept of dividing integers correctly. Explain clearly on dividing of integers and give brief notes.

**Synthesis:** Connecting division of integers to real-life.

**Evaluation:** Check if the learners can divide integers.





**Assessments:** Give learners exercises on division of integers and ensure that they can apply them to real-life. Give learners exercises on division of integers using learners' books such as Progress in Mathematics, Grade 8 (pages 17 - 18), Zambia Basic Education Course, Grade 8 (pages 63 - 65).



### Learning activity 7: Applying Combined Operations on Integers

**Introduction:** By now it is understandable that there are various operations that are used on integers in different situations. Learning combined operations on integers builds a strong foundation in Mathematics. It will help learners how to approach problems step by step, deciding which operation to apply first and simplify expressions efficiently using the operations; addition, subtraction, division and multiplication of integers.

**Problem Posing:** Simplify:  $(-2 \times -5) - (3 \times -4) = ?$

a) What is the final answer?

b) Why does multiplying two negative numbers give a positive result, and how does that affect subtraction here?



### Activity 7. 1: Research on Combined Operations

#### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial Environment:** Books
- **Natural Environment:** Objects in the environment.

- **Technological Environment:** Computers, Tablets, iPad, Calculators and Phones.

**What to do:** Put the learners in manageable groups of mixed abilities.

**Task:** Ask the learners to conduct some research on combined operations of integers and make presentations to the class. Let the other learners ask questions and choose the best presenters become winners.



**Content Tip:** Guide the learners to come up with the correct rules for the order of operations on combined integers.

**Skills Developed:** Teamwork, research and presentation skills.



### Activity 7. 2: Card Game

#### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial Environment:** Number cards.

**What to do:** Divide learners into small manageable groups. Distribute a card to each group.

**Task:** Explain the rules of the game to the learners that there are four (4) numbers shown on the card, use “+”, “-“, ”x”, “÷ ” Or ( ) to form a numerical expressions equal to 24. The four numbers to be used as directed numbers are 1, 2, 3 and 4. For example,  $(4 \div 2) + (-3 + 1) = 0$ ,  $-4 - (-3) \times 2 = 1$ . The group that forms more expressions in the shortest set time to get correct answers will be the winner. The teacher can limit the number of terms for quick discussions.

**Conceptualization:** Ensure the learners bring out the concept of applying combined operations on integers correctly. Explain clearly on combined operations on integers and give brief notes.

**Synthesis:** Connecting combined operations on integers to real-life situations.

**Evaluation:** Check if the learners can apply combined operations on integers in real-life.



**Assessments:** Give learners exercises on combined operations on integers and ensure they can apply them to real-life. Use learners’ books such as Progress in Mathematics, Grade 8 (pages 15 - 16), Zambia Basic Education Course, Grade 8 (pages 65 - 66).



## Learning Activity 8: Using ICT Tools to Enhance Understanding of Integers

### Introduction

The use of ICT tools can improve and reinforce learners' understanding with integers. There are various interactive tools that can be such as GeoGebra, IXL Math, Math Blaster-game-based learning and Kahoot.

**Problem Posing:** *Using a calculator or a coding tool like Python to evaluate this expression step by step :  $(-2+6) \times (-4) \div 2$ . Does the calculator follow the correct order of operation? Explain your findings.*

### Activity 8. 1: Integer War Game

#### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Technological Environment:** spreadsheet, GeoGebra and Desmos- online math games

**What to do:** Put the learners in small manageable groups. Ask them to practice integer operations using technology such as spreadsheets GeoGebra and Desmos (e.g.  $A1 + B1$ ,  $A1 * B1$ ,  $A1 / B1$ ; product  $(A1, B1)$ ).

**Task:** Use a spreadsheet, Desmos and GeoGebra to perform integers operations on addition, subtraction, division and multiplication by typing or entering sample integer values into columns A and B. Let them present their findings to the class and explain the logic.



**Skills to be developed:** Creativity, critical thinking, problem-solving, logical reasoning and ICT skills.

**Conceptualization:** Make sure that ICT tools are explored to cement learners' understanding.

**Synthesis:** Connect the use of integer operations to real- life situations.

**Evaluation:** Check the understanding of real–life scenarios on integer operations.



**Assessment:** Give learners a project-based assignment to create integer activities, scenarios and games using digital tools.

**Summative Assessment:** Investigative the how integers are applied in businesses

**Expected Standard:** By the end of this topic learners should: Four Operations applied on Integers accurately in real life.

### Summary

#### Key Points

- Explore integers in the environment
- Add integers
- Subtract integers
- Multiply integers
- Divide integers

## TOPIC 4: ALGEBRA

### Introduction

**Overview:** Algebra is a branch of Mathematics that deals with symbols and the rules for manipulating those symbols. It provides a framework for formulating equations, solving problems and representing relationships between variables.

In this topic, the sub-topic is algebraic expressions.

### General Competencies

- **Analytical Thinking:** Simplify complex real-life scenarios using algebraic expressions
- **Communication:** Ability to explain complex Algebraic concepts in an accessible manner
- **Critical Thinking:** Analyse word statements to come up with algebraic expressions.
- **Problem Solving:** Solve problems involving commutative, associative and distributive laws



### Key terms /Words/Vocabulary

- **Variables:** Symbols often letters that represent unknown values (*e. g. x, y*).
- **Constants:** Fixed values (*e. g. numbers like 2, -5* )
- **Expressions:** Combination of variables and constants (*e. g.  $3x + 4$* )
- **Factoring:** Rewriting an expression as a product of its factor
- **Exponent:** Raising a number to a power (*e. g.  $x^2$* )
- **Term:** A single mathematical expression that can be a number, a variable, or a combination of both multiplied together. Terms are building blocks of algebraic expressions.

## Sub-Topic: Algebraic Expressions

### Introduction

Algebraic expressions are mathematical phrases that include numbers, variables (letters that represent unknown values), and operations (such as addition, subtraction, multiplication, and division). For example,  $3x + 5$  and  $2y^2 - 4y + 7$  are algebraic expressions. Understanding algebraic expressions is crucial for higher-level mathematics topics, such as algebra, calculus, and statistics. It also enhances critical thinking and problem solving abilities, helping learners develop logical reasoning skills.

**Specific Competence:** Apply algebraic expressions in different life contexts.



### **Learning Activity 1: Constructing algebraic expressions from word statements.**

**Introduction:** Construction of algebraic expressions from word statements can be a rewarding experience for learners as it provides a mix of word problems for learners to translate into expressions. Several engaging activities can be explored to help learners understand algebra. Sound knowledge of the basics such as translation of word statements, building complexities, practice with examples, real-world application, review and reinforcements through quizzes and games would make the lesson more understandable and captivating.

**Problem posing:** *A bus was transporting passengers to town for shopping. Along the way, three passengers got off. How many passengers arrived in town?*

#### **Activity1.1: Algebraic Word Problems.**

##### **Suggested Teaching and Learning Materials to Set up Learning Environment**

##### **Artificial Environment: Cards with Word Problems**

**What to do:** Divide the class into small manageable groups of mixed abilities.

Assign each group with different types of word statements involving addition, subtraction, multiplication and division. Let the learners discuss and write their agreed statements. For example, If each apple costs  $x$  kwacha and you buy 3 apples, write an expression for the total cost.

**Task:** Each group discusses their statements and create algebraic expressions. Then, they come together in new groups where each member shares their solutions.

**Content Tips:** Ensure that learners understand that 'x+5' means 'add 5 to a variable x' or '5 more than x', 'a +b' means 'the sum of a and b' and so on.

**Skills to be developed:** Critical thinking, presentation and team work.

### Activity 1.2: Expression Matching Game

#### Suggested Teaching and Learning Materials to Set up Learning Environment

**Artificial environment:** Cards with word problems, algebraic expression cards

**What to do:** Prepare well in advance enough cards with word problems on them, and their corresponding algebraic expression cards. Put the learners in manageable groups, provide them with the cards so that they can do the matching game.

**Task:** Ask learners to perform the matching game. The group that finishes fast becomes the winner.



**Content Tip:** Word statements can also be written in form of symbols, for example, 'the product of a and b' can be written as ' $ab$ '. give learners such statements and their matching symbols. Some more examples are

- a) The sum of  $p$  and the square of  $q$
- b) Three times the product of  $a$  and  $b$
- c) Twice  $n$  minus  $m$
- d) The square of the sum of  $c$  and  $d$
- e) 10 less than  $c$



**Skills to be developed:** Collaboration, critical thinking and presentation

### Activity 1.3: Field Trip

#### Suggested Teaching and Learning Materials to Set up Learning Environment

**Artificial environment:** Data from local businesses

**What to do:** Put learners into small groups of varying abilities. Give them the task of gathering data on local pricing or statistics of various goods. Let them create algebraic expressions that model those situations.

**Task:** Learners take a field trip and collect data on the pricing or statistics on various goods. Let them create algebraic expressions and make presentations.



**Skills to be developed:** Inquiry, data collection, presentation, critical thinking.

**Conceptualization:** Make sure the concept of construction of algebraic expression from word statements is coming out accordingly with consideration of all the four operations. Demonstrate creation of algebraic expressions involving multiplication and division.

**Synthesis:** Connect construction of algebraic expressions to real-life scenarios.

**Evaluation:** Check if the learners can construct algebraic expressions from given statements. Use a check list to assess learners' understanding of the concept.

**Assessment:** Ask learners questions that depict real-world application of algebraic expressions. Give exercises from books such as Zambia Secondary School Syllabus book 10 (page 61), and Progress in Mathematics book 8 (page 48 - 49).

## **Learning Activity 2: Relating letters and numbers.**

### **Introduction**

Relating letters and numbers in algebra is important for several reasons; one of them is that letters (variables) allow us to represent general cases and patterns, making it easier to solve a wide range of problems without specifying particular values. Moreover, using letters encourages abstract reasoning, helping learners and mathematicians think about relationships and structures rather than just specific numbers. Teaching the relationship between letters and numbers in algebra can be made engaging through several practical activities. Some effective strategies are: variable scavenger hunt, Algebra tiles, card games with Algebraic expressions, hands on projects and many more.



**Problem Posing:** A shop sells  $x$  oranges at K2 each. Another shop sells the same number of oranges at K1.50 each but adds a K3 packaging fee. Write algebraic expressions for both shops and determine which is cheaper for  $x = 5$ .



### Activity 2.1: Variable Scavenger Hunt.

#### Suggested Teaching and Learning Materials to Set up Learning Environment

**Artificial Environment:** Empty bottles of different beverages, plastics, papers and any other materials that litter the environment.

**What to do:** Put learners in groups and guide them on how to proceed with the group task. Set boundaries by stating the area which learners can search for the described items.

**Task:** Let learners develop a list of items that they picked and represent them with different variables. Each item should have a corresponding variable and a brief description. Here is an example list.

Item Description	Variable
- Bottles of drinks collected that litter the surrounding	$d$
- Biscuits empty packs that litter the surrounding	$b$
- Plastic bags that litter the	$p$

surrounding	
- Leaves collected that litter the surrounding	<i>l</i>
etc.	

Let learners relate word statements to variables and constants.



**Content Tips:** Let learners formulate word statements and state clearly the variable in the statement and the constant e.g "Sarah has picked 3 empty bottles, and her friend gives her  $y$  more empty bottles. How many empty bottles does Sarah have now?"

**3** = Constant (fixed number of empty bottles Sarah has)

**y** = Variable (number of empty bottles her friend gives)

**Expression:**  $3+y$



**Skills to be developed:** Communication, collaboration and presentation

**Conceptualization:** Make sure the concept of relating letters (variables) and numbers to word statements is coming out clearly.

**Synthesis:** Relate variables and numbers to real-life scenarios.

**Evaluation:** Check if learners can relate variables and numbers to word statements.

**Assessment:** Ask learners to carry out a research on how variables help in understanding relationships between quantities and their use to everyday life. Give learners exercises from books such as Zambia Basic education course pupils book (page 67 - 68).



### Learning Activity 3: Identifying Variables and Coefficients

#### Introduction

Coefficients and variables are fundamental concepts used in expressions. Coefficients are numerical factors that multiply a variable. It represents a number of times the variable is counted. Variables are symbols for unknown or changeable values. Teaching variables and coefficients in algebra can be made engaging and practical through hands on activities that help learners understand concepts in a practical way.

**Problem Posing:** *Given the expression  $3y - 5$ , if  $y$  is doubled, what will be the new value of the expression? How does altering the variable  $y$  affect the overall expression, and what if instead, we halved  $y$ ?*



**Activity 3.1: Variable and Coefficient Identification**

**Suggested Teaching and Learning Materials to Set up Learning Environment**

**Artificial Environment: Worksheets**

**What to do:** Put learners into manageable groups and give the respective groups worksheets which have algebraic expressions such as;

Identify the, variables, coefficients and constants in the following expressions

Algebraic Expression	Description
a. $7m + 4$	Variable -  Coefficient -  Constant -
b. $-c + 10$	Variable -  Coefficient -

	Constant -
--	------------

**Task:** Let learners identify the variables and coefficient for the algebraic expressions given.



**Skills to be developed:** Collaboration and critical thinking



**Content Tips:** Explain to learners that variables are the letters that can change value. Coefficients are the numbers in front of the variables, showing how many times the variable is multiplied. If there is no number before a variable, the coefficient is 1. A negative sign in front of a number is part of the coefficient.

**Conceptualization:** Make sure that the concept of coefficient as a numerical factor, and variable as a representation of the unknown values in an algebraic expression is understood by learners.

**Synthesis:** Relate variables and coefficient to real-life scenarios.

**Evaluation:** Check if learners can relate variables and coefficients to word statements.

**Assessment:** Ask learners how variables and coefficients help in understanding relationships between quantities and their use to everyday life. Give learners exercises from books such as Zambia basic education course pupils book 8 (page 69).



#### **Learning Activity 4: Grouping Like terms and Unlike terms**

##### **Introduction**

Like terms are terms with the same variable raised to the same power, for example  $3x$  and  $5x$ ,  $y^3$  and  $-5y^3$ . Unlike terms are terms with different variables or different powers of the same variable, for example  $2x$  and  $3y$ ,  $y^3$  and  $-5y^2$ . Teaching like and unlike terms in algebra can be engaging and effective through a variety of practical activities.

**Problem Posing:** *Imagine you're helping organize a school fundraiser. You have two types of donation boxes: one for coins and another for notes. The coin box has contributions like  $3x$  Kwacha,  $5x$  Kwacha, and  $2y$  Kwacha, while the note box has  $4x$  Kwacha,  $7y$  Kwacha, and  $x$  Kwacha. To make accounting easier, you want to combine like terms for both boxes.*

*Can you identify and group the like terms from both boxes?*

*How would you explain the difference between like and unlike terms to a classmate using this example?*

*After combining like terms, what is the total amount collected in each box?*



#### **Activity 4.1: Real Life Scenario Algebraic Term Sorting**

##### **Suggested Teaching and Learning Materials to Set up Learning Environment**

**Artificial Environment:** Worksheet with real life scenarios

**What to do:** Put learners in groups of mixed abilities. Let them work out real life scenarios given on worksheets by grouping like terms.

Ask the learners why they have sorted out like terms in that order.

**Task:** Let learners work out the algebraic expressions by grouping like and unlike terms. For example, you are designing a garden with different types of plants. The number of flower plants is represented by  $4a + 3b$ , and the number of vegetable plants is  $2a + 5b + a$ .

- a. Can you identify and combine the like terms to find the total number of each type of plant?
- b. Which terms in the expressions are unlike, and why can't they be combined?



**Skills to be developed:** Analytical thinking, communication and collaboration.

**Conceptualization:** Make sure the concept of like and unlike terms is understood by learners.

**Synthesis:** Relate like and unlike terms to real-life scenarios

**Evaluation:** Check if learners can work with like and unlike terms using open-ended questions. Get feedback such as what strategies did you use to identify like terms? Were there any tricky expressions? How can recognizing like and unlike terms help in solving real-life problems?



**Assessment:** Ask learners to give more real-life examples on like and unlike terms. Give learners exercises from books such as Zambia Basic Education Course Pupils Book 8 (pages 71 - 72).

### **Learning activity 5: Expanding and Simplifying Algebraic Expressions**

#### **Introduction**

The expansion and simplification of algebraic expressions is crucial because it facilitates understanding, enhances problem solving and prepares for advanced concepts among others.

**Problem Posing:** *A carpenter is designing a rectangular table with a side length of  $(x + 2)$  metres and width 7 metres. Write an algebraic expression for the area of the table and simplify it.*



#### **Activity 5.1: Treasure Hunt**

#### **Suggested Teaching and Learning Materials to Set up Learning Environment**

- **Artificial Environment:** Cards with original terms/expressions in expanded form, cards with simplified form.

**What to do:** Prepare original algebraic expression cards and stick them under their desks, not simplified and the corresponding algebraic expression card, which are simplified stuck on the wall. Learners should be in small manageable groups of mixed abilities. Tell the learners that “*You are treasure hunters in a math adventure! To move from one clue to the next, you need to expand and simplify the algebraic expressions correctly. The first clue is hidden under your desk.*”

**Task:** Learners work in groups to find the expression, expand and simplify it. Let them match the original expressions with their simplified counterparts on the wall. Discuss as a class afterwards.



**Content Tips:** Let learners have expressions such as  $3(x+2) + 2(2x+5)$  with  $7x+16$  stuck on the wall together with other solutions for other expression. Observe individual learners in their groups whether they are cooperative and whether they collaborate with their colleagues.



**Skills to be developed:** Collaboration and communication

### **Activity 5.2: Expanding and Simplifying Expressions Using Online Tools**

#### **Suggested Teaching and Learning Materials to Set up Learning Environment**

- **Technological Environment:** Computers, projectors, phones

**What to do:** Put learners in small manageable groups with mixed abilities and allow them to have access to digital gadgets. Use online tools to practice expanding and simplifying algebraic expressions. Encourage learners to share their strategies or tips that they discover while using the tools.

**Task:** Learners use online tools to practice expansion and simplification of algebraic expressions. Learners should share their discoveries as they use the tools and project their presentations.



**Skills to be developed:** Analytical thinking, collaboration, communication and ICT skills.

**Content tip:** Make sure that learners are given algebraic expression questions of the form  $a(bx - c) - (2abx - 3ac)$  for them to simplify. Use suitable sites like khan academy for better and quality video content.

**Conceptualization:** Ensure that the concept of expanding and simplifying algebraic expressions is applied in real-life.

**Synthesis:** Connect the expansion and simplification of algebraic expressions to real-life.

**Evaluation:** Observe the learners in their groups as they carry out the activities. Find out the learners' attitude towards the group work and integrity. Check if they are able to do the work in harmony. During the process of the description or exploration learners should communicate effectively. Are they learning from one another? are they creative? Check if learners can expand and simplify algebraic expressions and use them in their daily lives.



**Assessment:** Let learners investigate how expansion and simplification of algebraic expressions can help in understanding relationships between quantities and their use to everyday life. Give exercises such as the ones on pages 72 - 74 of Zambia Basic Education Course Mathematics Pupils Book 8, exercises 1, 2, 3 and 4, Zambia Basic Education Course Mathematics Pupils' Books 9, pages 93 - 97.



### **Learning activity 6: Applying Commutative, Associative and Distributive (CAD) Laws**

#### **Introduction**

The commutative, distributive and associative laws are fundamental concepts in algebra that describe how numbers and variables interact. Commutative law states that the order of numbers/variables does not affect the result of an operation. This applies to both addition and multiplication. The law allows for flexibility in rearranging terms. The distributive law connects addition and multiplication, showing how to distribute multiplication over addition. This property is crucial for simplifying expressions and solving equations, as it allows for the expansion of terms. Associative law indicates that the way numbers are grouped does not change the result of the operation. This law also applies to both addition and multiplication. The law helps in restructuring expressions to make calculations more manageable.

These laws are fundamental in algebra because they enable the manipulation of expressions and equations. They are essential in various aspects of mathematics, including solving equations, factoring and simplifying expressions. Understanding these laws is key to progressing in algebra and higher mathematics.



**Problem Posing:** *A shopkeeper loves playing number tricks with his customers. One day he tells a customer “I sell baskets of fruits in a magical way. If you buy 4 apples and 6 bananas, then triple the order, you count in different ways but always get the same results. Can you prove my magic?”*

### Activity 6.1: Commutative law

#### Suggested Teaching and Learning Materials to Set up the Learning Environment

➤ **Artificial Environment:** Manila paper

**What to do:** Put learners into small manageable groups. Observe the learners in their groups as they carry out the tasks below. During the process, are learners communicating effectively? Are they learning from one another? Are they creative?

**Task:** Ask learners to cut the manila paper into squares or rectangular shapes. Let learners assign variables to the square or rectangular paper cuts e.g.  $x$  for length and  $y$  for breadth/width. Ask learners to calculate the area of a square or rectangular paper cut. Let them rotate the square or rectangular paper cuts so that  $x$  now becomes the width/breadth and  $y$  becomes the length. Let them calculate the area again of the square or rectangular paper cut. Ask them to compare and discuss the two results.



**Skills to be developed:** Analytical thinking, collaboration and communication.

### Activity 6.2: Associative Law

#### Suggested Teaching and Learning Materials to Set up the Learning Environment

**Artificial Environment:** Term cards/Expression card

**What to do:** Put learners in small manageable groups. Provide learners with term cards/expression cards such as  $2x$ ,  $3x$  and  $5x$ .

**Task:** Ask learners to arrange the cards in different groupings for addition and multiplication. For Addition:  $(2x + 3x) + 5x$ ,  $(3x + 5x) + 2x$  and  $(2x + 5x) + 3x$ .

For Multiplication:  $(2x \times 3x) \times 5x$  ,  $(3x \times 5x) \times 2x$  and  $(2x \times 5x) \times 3x$  . Ask learners to find answers for the addition and multiplication, and compare the results in each case.



**Content Tip:** Ensure that learners work with all the CAD laws on algebraic expressions.



**Skills Developed:** Analytical thinking, collaboration and communication.

### Activity 6.3: Distributive Law

#### Suggested Teaching and Learning Materials to Set up the Learning Environment

**Artificial Environment:** Term cards/Expression card

**What to do:** Put learners in small manageable groups. Give learners a scenario such as *your school has decided to make a garden. You need to buy flowers and manure. The cost of one flower is represented by  $(x+5)$  kwacha, where  $x$  is the cost of plants and  $K5$  is the cost of plastic for putting the plant. The manure costs  $(y+3)$  Kwacha per kilogram, where  $y$  is the cost of manure and  $K3$  is for packaging. You are tasked with calculating the total cost for 4 flowers and 3 kilograms of manure.*

**Task:** Ask learners to formulate the Expression and write an expression for the total cost such as  $4(x + 5) + 3(y + 3)$ . Guide them to use the distributive law to expand the expression  $4(x + 5)$  to get  $4x + 20$  and  $3(y + 3)$  to get  $3y + 9$ . Combine the expanded terms to get the total cost  $4x + 20 + 3y + 9$ , and group the like terms to get  $4x + 3y + 29$ .



**Content Tip:** Let learners write a short reflection on **how** understanding the distributive law helped them solve a real-world problem and **why** it is important in everyday life (e.g., budgeting, shopping, planning events). This activity ensures that learners engage with the distributive law in a meaningful and practical way, developing both mathematical understanding and real-life problem-solving skills.



**Skills to be developed:** Analytical thinking, problem solving, collaboration and communication.



## Learning Activity 7: Factorizing Algebraic Expressions Using Common Factors

### Introduction

Factorizing algebraic expressions is a fundamental skill in algebra that involves breaking down complex expressions into simpler, multiplicative components. Factorizing algebraic expressions by common factors is one of the most basic and widely used methods. This method is particularly useful when all terms in an expression share a common factor, which can be factored out to simplify the expression.

**Problem Posing:** *Your school is planning a cultural event, and you are part of the organizing committee. You need to buy decorations and snacks. The total cost of decorations and snacks can be represented by the expression:  $2x+4xy$ , where  $x$  represents the cost of each decoration, and  $y$  represents the number of snack packs per decoration. To simplify budgeting, you need to factorize the expression to understand the cost structure better. Factorise the expression.*

### Activity 7.1 Error Analysis in Factorisation

#### Suggested Teaching and Learning Materials to Set up the Learning Environment

- **Artificial Environment:** Worksheets

**What to do:** Put learners in small manageable groups. Provide them with incorrectly factored expressions on the worksheets.

**Task:** Ask the learners to identify the mistakes and correct them.



**Skills Developed:** Collaboration, communication, critical thinking and problem solving.

## Activity 7.2: Factorisation Treasure Hunt

### Suggested Teaching and Learning Materials to Set up the Learning Environment

- **Artificial Environment:** Expression cards and Factor cards

**What to do:** Put learners in small manageable groups. Place number cards around the classroom or outdoor area.

**Task:** Let each group pick an expression card and search for its common factors among the factor cards placed around. Once the group collects the correct factor, they must write the factorization of their expression.

**Content Tip:** For numbers you can revise with the learners on how to find the HCF which is useful for factorisation.



**Skills to be developed:** Collaboration, communication, critical thinking and problem solving.

**Conceptualization:** Make sure the concept of factorizing algebraic expressions by common factors is applied in real-life situation.

**Synthesis:** Connect the factorization of algebraic expressions by common factors to real-life

**Evaluation:** Check if learners can factorize algebraic expression by common factor and use them in their day today life. Give them problem-posing tasks which engages learners in factorisation but also encourages them to think critically about how mathematics applies to real-life situations.



**Assessment:** Give exercises using books such as Progress in Mathematics Learners Book 8 on page 53, Progress in Mathematics Learners Book 10. Page 33 and Zambia Senior Secondary Syllabus Mathematics Grade 10 Pupils Book, page 70

### Learning Activity 8: Applying the Four Operations on Algebraic Expressions.

#### Introduction

Applying the four operations (addition, subtraction, multiplication and division) to algebraic expressions can be made practical and engaging through various activities such as building blocks, manipulating real life scenarios, group work, digital tools and budgeting.

**Problem Posing:** *Why does repeatedly multiplying an algebraic expression by a fraction or decimal lead to rapid decrease in its value?*

### Activity 8. 1: Four Operations on Algebraic Expressions

#### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial Environment:** Worksheets

**What to do:** Make worksheets for learners to use in groups.

**Task:** Work out questions given from the work sheets involving all the four operations on Algebraic expressions such as work out a.  $2x - y + xy - 4(x + 3y) - 5xy$ ,  $5ab + a - 2(4a \div a)$



**Skills developed:** Collaboration, communication and critical thinking.

### Activity 8. 2: The Four Operations.

#### Suggested Teaching and Learning Materials to Set up Learning Environment

- **Artificial Environment:** Worksheets.

**What to do:** Put the learners in groups of mixed abilities. Provide them with work sheets to answer questions.

**Task:** Lead the learners into applying the four operations on algebraic expressions. Present them with questions on real life situations on the work sheets such as finding the area, perimeter, or financial applications e.g.

- Find the area of the rectangular garden of length  $(2x + 3)m$  and of width  $ym$ .
- Find the concentration if a solution has  $2x + 4$  moles in a volume of  $2x$  litres
- Each glass of lemonade costs  $x+2x$  Kwacha to make. You sell  $4x$  glasses. Write an expression for the total cost. Simplify the expression.

- Your total profit from jewelry sales is  $8x^2 + 16x$ . You split the profit equally among 4 team members. How much does each person get?
- The cost of decorations is  $3x + 5$  and the cost of food is  $2x + 8$ . What is the total cost? If you return decorations worth  $x + 2$ , what is the new total cost?



**Skills to be developed:** Collaboration, communication and problem solving.

**Conceptualization:** Make sure the application of the four operations on algebraic expressions comes out correctly.

**Synthesis:** Connect the use of the four operations on algebraic expressions to real-life situations.

**Evaluation:** Observe the learners in their groups as they carry out the activities. Find out the learners' attitude towards the group work and integrity. Check if they are able to do the work in harmony. During the process of the description or exploration learners should communicate effectively. Check if they are able to learn from one another and if they are creative. Check if learners can use the four operations on algebraic expressions and apply them in their daily lives.



**Assessment:** Give the learners exercises to practice using materials such as Progress in Mathematics, Grade 8 (pages 57 - 58), Zambia Basic Education Course Grade 8 (pages 102 - 103) or any other helpful resource.



### **Learning Activity 9: Evaluating Algebraic Expressions by Substituting Variables with Numbers**

#### **Introduction**

Substitution in algebra is a fundamental concept where variables are replaced with specific values to simplify expressions. This helps learners grasp the concept that letters in algebra stand for numbers and can be replaced accordingly.

**Problem Posing:** *Imagine you are an event planner tasked with organizing a school concert. The total cost of the event depends on the number of performers ( $p$ ) and the hours ( $h$ ) of rehearsal. Renting the venue is a fixed cost of 100. The cost expression is given by:  $50p+20h+100$ . If there are 8 performers and 5 hours of rehearsal, what was the total cost?*

### **Activity 9.1: Evaluating of Algebraic Expressions by Substitution.**

#### **Suggested Teaching and Learning Materials to set up the Learning Environment**

- **Artificial Environment:** Worksheets

**What to do:** Design worksheets for learners with word problems such as

(Calculating phone credit balance. Joy has K20 in prepaid phone credit. Each call costs K2 and each text message costs K0.50. If she make  $x$  calls and sends  $y$  texts, the remaining balance is given by the expression:  $20 - 2x + 0.5y$ )

**Task:** Let learners substitute different values of  $x$  and  $y$  to see how long her of credit lasts.

Content Tips: Design a worksheet with different questioning techniques.



**Skills to be developed:** Analytical thinking, Collaboration, Communication, Critical thinking and Problem solving.

### **Activity 9.2: Class Competition on Algebra**

#### **Suggested Teaching and Learning Materials to set up the Learning Environment**

- **Artificial Environment:** Work Cards

**What to do:** Organize a game where learners earn points for correctly Substituting and evaluating expressions. Provide learners with work Cards and let them work out the questions. The group which scores the highest becomes winners.

**Task:** The learners to work out the questions on work card.



**Skills to be developed:** Collaboration, Computation fluency and Problem-solving and teamwork.

**Conceptualization:** Make sure the concept of substitution in algebra is coming out clearly. Explain clearly the concept of substitution and give brief notes.

**Synthesis:** Relate Substitution in Algebra to real-life situation

**Evaluation:** Check if the learners can correctly evaluate algebraic expressions by substitution. Ask learners open ended questions to confirm grasping of the concept.



**Assessment:** Give learners' exercises on evaluating algebraic expression by Substitution. Use books such as Zambia Secondary Basic Education Course (Page 74 - 75), and Progress in Mathematics (Page 54).

**Summative Assessment:** Identify a problem in your community and suggest how best the problem can be solved by identifying companies who can come on board to help. Use algebraic expression in the situation identified.

**Expected Standards:** Algebraic expressions applied correctly in different life context

**Topic Evaluation:**

- 1) If a recipe requires  $(2x + 1)$  cups of flour and you want to make  $(x + 3)$  times the recipe, what would be the total flour needed?
- 2) If a building requires  $(4x + 10)$  bricks for each wall and there are  $(x + 2)$  walls, find an expression for the total number of bricks.
- 3) The car battery sells for  $(20 - x)$  kwacha and was bought for  $(5 + x)$  kwacha. Find the profit when sold.
- 4) If a car covered a distance of  $2x$  km at a speed of  $(3x + 5)$  km/hr, Find the time spent to cover the whole journey.
- 5) The length of a rectangle is  $(x + 5)$  cm, and its breath is 6cm. Write an expression in  $x$  for the area of the rectangle.
- 6) A collection of coins consisted of  $3x$  five ngwee coins  $(x + 2)$  fifty ngwee coins and  $(x - 6)$  ten ngwee coins. Find in its simplest form an expression for the amount of the collection in ngwee.
- 7) If a town's population is represented by  $(1000 + 50x)$ , where  $x$  is the number of years. How many people would be there in 15 years' time.



- 8) The expression  $110 + \frac{A}{2}$  is used to estimate a person's blood pressure. In this expression A stand for the person's age in years.  
Estimate the blood pressure for a person who is 15 years old.
- 9) Write two different expressions that mean the same as  $2xy$ .
- 10) Evaluate the expression  $\frac{9(x+y)}{3z}$  if  $x = 1, y = 3$  and  $z = 2$
- 11) Explain in your own language how variables are used.
- 12) When scientists dilute acid, they always pour the acid into water. Pouring water into acid could produce spattering and burns.  
These two actions are not commutative. Give two examples from your life of actions that are not commutative.

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